CSC 456/656 Fall 2025 Study for Third Examination April 9, 2025

1. Review **tf1ans.pdf**, **tf2ans.pdf** and **tf3ans.pdf**.

2. Use the pumping lemma for regular languages to prove that $L = \{a^n b^n\}$ is not regular.

3. Use the pumping lemma for context-free languages to prove that $L = \{a^n b^n c^n\}$ is not context-free.

4. Prove that the complement of $\{a^n b^n c^n\}$ is context-free. (Hint: A CF grammar for that language has lots of productions.)

5. Prove that the set of recursive real numbers is countable.

6. Prove the theorems on the handout CanonEnum.pdf.

7. Review the proof that the halting problem is undecidable.

8. What is the Church-Turing thesis?

9. Give a definition of the class $\mathcal{NC}$.

10. Give a definition of the class $\mathcal{P}$–complete.

11. Name a language which is known to be $\mathcal{P}$–complete.

12. Give a definition of the class $\mathcal{P}$–SPACE–complete.

13. Name a language which is known to be $\mathcal{P}$–SPACE–complete.

14. Let L be the Dyck language, but where each left parenthesis is written as a and every right parenthesis as b. (This makes grading easier, since if you write parentheses carelessly, they look alike.)

    Here is an unambiguous CFG for L.

    1. $S \rightarrow a_2 S_3 b_4 S_5$
    2. $S \rightarrow \lambda$

    (a) Fill in the action and goto tables for the grammar given above. I have started the tables by writing row 0 and row 4.

    (b) Show the computation of the parser for the input string $aabbab$.

	a	b	$\$$	S
0	s2		r2	1
1				
2				
3				
4	s2	r2	r2	5
5				

15. The following CF grammar models an assignment statement. We allow just two identifiers, x and y, and two operators $+$ and $*$. We have three grammar variables, S for assignment statement, I for identifier, and E for expression. We have the equal sign as a symbol. The start symbol is S.

1. $S \rightarrow I_2 =_3 E_4$
2. $I \rightarrow x_5$
3. $I \rightarrow y_6$
4. $E \rightarrow I_7$
5. $E \rightarrow E +_8 E_9$
6. $E \rightarrow E *_{10} E_{11}$

(a) Sketch the parse tree of the string
$$x = y + x * y$$

	x	y	$+$	$*$	$=$	$\$$	S	I	E
0	s5	s6					1	2	
1						halt			
2					s3				
3	s5	s6						7	4
4			s8	s10		r1			
5			r2	r2	r2	r2			
6			r3	r3	r3	r3			
7			r4	r4	r4	r4			
8	s5	s6						7	9
9			r5	s10		r5			
10	s5	s6						7	11
11			r6	r6		r6			

(b) Identify the entries of the Action table which ensure that addition and multiplicaion are left associative and that multiplication has precedence over addition.

16. (a) Give a CNF grammar for the language L of problem 14.
    (b) Use that grammar and the CYK algorithm to prove that $aababb \in L$.

17.