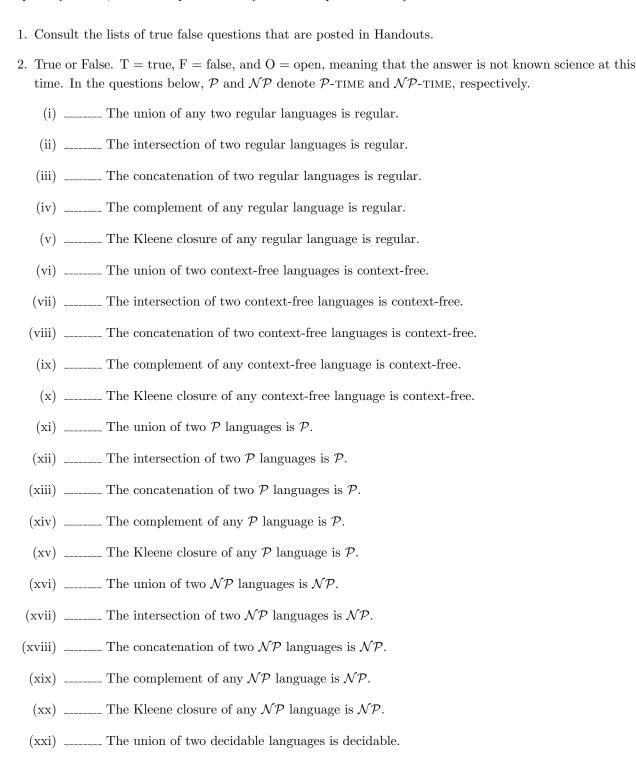
University of Nevada, Las Vegas Computer Science 456/656 Spring 2025

Practice Problems for the Final Examination on May 14, 2025

Throughout, \mathcal{P} means \mathcal{P} -TIME.

Despite my efforts, this list of problems may contain duplicates. Sorry about that!



- (xxii) _____ The intersection of two decidable languages is decidable.
- (xxiii) _____ The concatenation of two decidable languages is decidable.
- (xxiv) _____ The complement of any decidable language is decidable.
- (xxv) _____ The Kleene closure of any decidable language is decidable.
- (xxvi) _____ The union of two \mathcal{RE} languages is \mathcal{RE} .
- (xxvii) _____ The intersection of two \mathcal{RE} languages is \mathcal{RE} .
- (xxviii) _____ The concatenation of two \mathcal{RE} languages is \mathcal{RE} .
- (xxix) _____ The complement of any \mathcal{RE} language is \mathcal{RE} .
- (xxx) _____ The Kleene closure of any \mathcal{RE} language is \mathcal{RE} .
- (xxxi) _____ The union of two undecidable languages is undecidable.
- (xxxii) _____ The intersection of two undecidable languages is undecidable.
- (xxxiii) _____ The complement of any undecidable language is undecidable.
- (xxxiv) _____ The Kleene closure of any undecidable language is undecidable.
- (xxxv) _____ The union of any \mathcal{NC} languages is \mathcal{NC} .
- (xxxvi) _____ The intersection of two \mathcal{NC} languages is \mathcal{NC} .
- (xxxvii) _____ The concatenation of \mathcal{NC} languages is \mathcal{NC} .
- (xxxviii) _____ The complement of any \mathcal{NC} language is \mathcal{NC} .
- (xxxix) _____ The Kleene closure of any \mathcal{NC} language is \mathcal{NC} .
 - (xl) $\mathcal{NC} = \mathcal{P}$.
 - (xli) _____ \mathcal{NC} . = \mathcal{P} -space.
 - (xlii) $\mathcal{P} = \mathcal{NP}$.
 - (xliii) $\mathcal{P} = \mathcal{P}$ -space.
 - (xliv) $\mathcal{P} = \text{EXP-TIME}$.
 - (xlv) ____ \mathcal{P} -SPACE = EXP-SPACE.
 - (xlvi) _____ There is a PDA that accepts the language of all palindromes over $\{a, b\}$.
- (xlvii) _____ If every $w \in L$ can be proved to be in L, then L must be decidable.
- (xlviii) _____ There is some PDA that accepts L, where L is the language over $\{a, b, c\}$ consisting of all strings which have more a's than b's and more b's than c's.

- (xlix) _____ The language $\{a^nb^n \mid n \geq 0\}$ is context-free.
 - (1) _____ The language $\{a^nb^nc^n \mid n \geq 0\}$ is context-free.
 - (li) _____ The language $\{a^ib^jc^k\mid j=i+k\}$ is context-free.
 - (lii) _____ The intersection of any regular language with any context-free language is context-free.
- (liii) _____ If L is a context-free language over an alphabet with just one symbol, then L is regular.
- (liv) _____ The set of strings that your high school algebra teacher would accept as legitimate expressions is a context-free language.
- (lv) _____ Every language accepted by a non-deterministic machine is accepted by some deterministic machine.
- (lvi) _____ The problem of whether a given string is generated by a given context-free grammar is decidable.
- (lvii) _____ Every language generated by an unambiguous context-free grammar is accepted by some DPDA.
- (lviii) _____ The language $\{a^nb^nc^n \mid n \geq 0\}$ is in the class \mathcal{P} -TIME.
- (lix) _____ There exists a polynomial time algorithm which finds the factors of any positive integer, where the input is given as a binary numeral.
- (lx) _____ Every \mathcal{NP} language is decidable.
- (lxi) $\mathcal{NP} = \mathcal{P}$ -SPACE
- (lxii) $_$ EXP-TIME = EXP-SPACE
- (lxiii) _____ The traveling salesman problem (TSP) is \mathcal{NP} -complete.
- (lxiv) _____ The knapsack problem is \mathcal{NP} -complete.
- (lxv) _____ The language consisting of all satisfiable Boolean expressions is \mathcal{NP} -complete.
- (lxvi) _____ The Boolean Circuit Problem is in \mathcal{P} .
- (lxvii) _____ The Boolean Circuit Problem is in \mathcal{NC} .
- (lxviii) $\underline{\hspace{1cm}}$ 2-SAT is \mathcal{P} -TIME.
- (lxix) _____ 3-SAT is $\mathcal{P}\text{-time}$.
- (lxx) _____ Primality, using binary numerals, is \mathcal{P} -TIME.
- (lxxi) _____ Every context-free language is in \mathcal{P} .
- (lxxii) _____ Every context-free language is in \mathcal{NC} .

(lxxiii) _____ Every language generated by an unrestricted grammar is recursive. (lxxiv) _____ The problem of whether two given context-free grammars generate the same language is decidable. (lxxv) _____ The language of all fractions (using base 10 numeration) whose values are less than π is decidable. (lxxvi) _____ There exists a polynomial time algorithm which finds the factors of any positive integer, where the input is given as a unary ("caveman") numeral. (lxxvii) _____ For any two languages L_1 and L_2 , if L_1 is undecidable and there is a recursive reduction of L_1 to L_2 , then L_2 must be undecidable. (lxxviii) _____ If P is a mathematical proposition that can be written using a string of length n, and P has a proof, then P must have a proof whose length is $O(2^{2^n})$. (lxxix) _____ If L is any \mathcal{NP} language, there must be a \mathcal{P} -TIME reduction of L to the partition problem. (lxxx) _____ Every bounded function is recursive. (lxxxi) _____ If L is \mathcal{NP} and also co- \mathcal{NP} , then L must be \mathcal{P} . (lxxxii) _____ If a language L is in \mathcal{RE} and also in co- \mathcal{RE} , then L must be decidable. (lxxxiii) _____ Every language is enumerable. (lxxxiv) _____ If a language L is undecidable, then there can be no machine that enumerates L. (lxxxv) _____ There is a non-recursive function which grows faster than any recursive function. There exists a machine that runs forever and outputs the string of decimal digits of π (the well-known ratio of the circumference of a circle to its diameter). (lxxxvii) _____ Rush Hour, the puzzle sold in game stores everywhere, generalized to a board of arbitrary size, is \mathcal{NP} -complete. (lxxxviii) _____ There is a polynomial time algorithm which determines whether any two regular expressions are equivalent. (lxxxix) _____ If two regular expressions are equivalent, there is a polynomial time proof that they are equivalent. (xc) _____ Every subset of a regular language is regular. (xci) _____ Let L be the language over $\{a,b,c\}$ consisting of all strings which have more a's than b's and more b's than c's. There is some PDA that accepts L. (xcii) _____ Every subset of any enumerable set is enumerable. (xciii) _____ If L is a context-free language which contains the empty string, then $L \setminus \{\lambda\}$ must be contextfree.

(xciv) _____ The computer language C++ has Turing power. (xcv) _____ Let Σ be the binary alphabet. Every $w \in \Sigma^*$ which starts with 1 is a binary numeral for a positive integer. Let $Sq: \Sigma^* \to \Sigma^*$ be a function which maps the binary numeral for any integer n to the binary numeral for n^2 . Then Sq is an \mathcal{NC} function. (xcvi) _____ If L is any \mathcal{P} -TIME language, there is an \mathcal{NC} reduction of L to the Boolean circuit problem. (xcvii) _____ If an abstract Pascal machine can perform a computation in polynomial time, there must be some Turing machine that can perform the same computation in polynomial time. (xcviii) _____ The binary integer factorization problem is co- \mathcal{NP} . (xcix) _____ Every subset of a regular language is decidable. (c) _____ Every language accepted by a non-deterministic machine is accepted by some deterministic machine. (ci) _____ The independent set problem is \mathcal{P} -TIME. (cii) _____ If S is a set of positive integers whose set of binary numerals decidable, then $\sum_{n \in S} 2^{-n}$ must be a recursive real number. (ciii) _____ Multiplication of matrices with binary numeral entries is \mathcal{NC} . (civ) _____ Equivalence of regular expressions is decidable. (cv) _____ Equivalence of context-free grammars is $co-\mathcal{RE}$. (cvi) _____ The language consisting of all fractions whose values are less than the natural logarithm of 5.0 is recursive. (cvii) _____ Every sliding block problem is \mathcal{P} -SPACE. (cviii) _____ There are uncountably many co- \mathcal{RE} languages. (cix) _____ If L is any \mathcal{P} -TIME language, there is an \mathcal{NC} reduction of L to CVP, the Boolean circuit problem. (cx) _____ Every finite language is regular. (cxi) _____ If L is a \mathcal{P} -TIME language, there is a Turing Machine which decides L in polynomial time. (cxii) ______ If anyone ever finds a polynomial time algorithm for any \mathcal{NP} -complete language, then we will know that $\mathcal{P} = \mathcal{NP}$. (cxiii) _____ RSA encryption is believed to be secure because it is believed that the factorization problem for integers is very hard. (cxiv) \perp If S is a set of positive integers whose set of binary numerals is recursively enumerable, then $\sum_{n \in S} 2^{-n}$ must be a recursive real number. (Hint: This problem is very hard.)

(cxv) _____ There is some PDA that accepts $\{w \in \{a,b,c\}^* : \#_a(w) > \#_b(w) > \#_c(w)\}$, that is, the language over $\{a, b, c\}$ consisting of all strings which have more a's than b's and more b's than c's. (cxvi) _____ If L is \mathcal{RE} and $w \in L$, there is a proof that $w \in L$. (cxvii) _____ Every language accepted by a non-deterministic machine is accepted by some deterministic machine. (cxviii) _____ The set of palindromes over $\{a,b\}$ is accepted by some DPDA. (cxix) _____ The language $\{a^nb^nc^n \mid n \geq 0\}$ is in the class \mathcal{NC} . (cxx) _____ Every problem that can be mathematically defined has an algorithmic solution. (cxxi) $\mathcal{NC} = \mathcal{P}$. (cxxii) $\mathcal{P} = \mathcal{NP}$. (cxxiii) _____ The set of binary numerals for prime numbers is \mathcal{P} -TIME. (cxxiv) _____ There is a gathematical proposition that is true but cannot be proved true. (cxxv) _____ The binary integer factorization problem is co- \mathcal{NP} . (cxxvi) _____ If L is \mathcal{NP} , there is a polynomial time reduction of L to the subset sum problem. (cxxvii) _____ The intersection of any two \mathcal{NP} languages is \mathcal{NP} . (cxxviii) _____ The intersection of any two co- \mathcal{NP} languages is co- \mathcal{NP} . (cxxix) _____ The intersection of any two co- \mathcal{RE} languages is co- \mathcal{RE} . (cxxx) _____ Multiplication of matrices with binary numeral entries is \mathcal{NC} . (cxxxi) _____ Every recursively enumerable language is generated by an unrestricted (general) grammar. (cxxxii) _____ Equivalence of context-free grammars is $co-\mathcal{RE}$. (cxxxiii) _____ The language of all true mathematical statements is recursively enumerable. (cxxxiv) _____ The language of all **provably** true mathematical statements is recursively enumerable. (cxxxv) _____ There are uncountably many undecidable languages over the binary alphabet. (cxxxvi) _____ RSA encryption is accepted as secure by most experts, because they believe that the factorization problem for binary numerals is very hard. (cxxxvii) _____ The language of all $\langle G_1 \rangle \langle G_2 \rangle$ such that G_1 and G_2 are CF grammars which are **not** equivalent is \mathcal{RE} .

(cxxxviii) _____ A real number x is recursive if and only if the set of fractions whose values are greater than

x is recursive (decidable).

(cxxxix)	For any real number x , there exists a machine that runs forever and outputs the string of decimal digits of x .
(cxl)	If a Boolean expression is satisfiable, there is a \mathcal{P} -TIME proof that it's satisfiable.
(cxli)	If there is a solution to a given instance of any sliding block problem, there must be a solution of polynomial length.
(cxlii)	If the Boolean circut problem (CVP) is \mathcal{NC} , then $\mathcal{P} = \mathcal{NC}$.
3. Give	a definition of each term.
(i)	Accept. (That is, what does it mean for a machine to accept a language.)
(ii)	Decide. (That is, what does it mean for a machine to decide a language.)
(iii)	Canonical order of a language L .
(iv)	Give the verifier-certificate definition of the class \mathcal{NP} .
(v)	State the pumping lemma for regular languages.
(vi)	State the pumping lemma for context-free languages.
(vii)	What is the importance nowdays of \mathcal{NC} ?
(viii)	State the Church-Turing thesis.
4. Whi	ch class of languages does each of these machine classes accept?
(i)	Deterministic finite automata.
(ii)	Non-deterministic finite automata.
(iii)	Push-down automata.
(iv)	Turing Machines.

- 5. The LALR parser given for this grammar:
 - 1. $E \to E_{-2} E_3$
 - 2. $E \rightarrow E *_4 E_5$
 - 3. $E \rightarrow x_6$

contains errors, meaning that it might parse a string in a manner that would be considered incorrect by your programming instructor. Find those errors and correct them.

	x	_	*	\$	$\mid E \mid$
0	s6				1
1		s2	s4	halt	
2	s6				3
3		r1	r1	r1	
4	s6				5
5		s2	s4	r2	
6		r3	r3	r3	

- 6. Every language, or problem, falls into exactly one of these categories. For each of the languages, write a letter indicating the correct category.
 - **A** Known to be \mathcal{NC} .
 - **B** Known to be \mathcal{P} -TIME, but not known to be \mathcal{NC} .
 - C Known to be \mathcal{NP} , but not known to be \mathcal{P} -TIME and not known to be \mathcal{NP} -complete.
 - **D** Known to be \mathcal{NP} -complete.
 - **E** Known to be \mathcal{P} -SPACE but not known to be \mathcal{NP}
 - **F** Known to be EXP-TIME but not nown to be \mathcal{P} -SPACE.
 - **G** Known to be EXP-SPACE but not nown to be EXP-TIME.
 - H Known to be decidable, but not nown to be EXP-SPACE.
 - I \mathcal{RE} but not decidable.
 - \mathbf{K} co- \mathcal{RE} but not decidable.
 - **L** Neither \mathcal{RE} nor co- \mathcal{RE} .
 - (i) _____ All C++ programs which halt with no input.
 - (ii) _____ All base 10 numerals for perfect squares.
 - (iii) _____ All configurations of RUSH HOUR from which it's possible to win.
 - (iv) _____ All satisfiable Boolean expressions.
 - (v) _____ All binary numerals for composite integers. (Composite means not prime.)
 - (vi) _____ The furniture mover's problem.
 - (vii) _____ The set of all positions of Chinese GO, on a board of any size, from which white can win.
 - (viii) _____ All C++ programs which do not halt if given themselves as input.
 - (ix) _____ The Dyck language.

(x)	The Jigsaw problem. (That is, given a finite set of
	assembled into a rectangle, with no overlap and no spaces.)
(xi)	Factorization of binary numerals.
(xii)	SAT.
(xiii)	3-SAT.
(xiv)	2-SAT.
(xv)	The Independent Set problem.
(xvi)	The Subset Sum Problem.
(xvii)	The block sorting problem.
(xviii)	The sliding block problem.
(xix)	The Hamiltonian cyle problem.
(xx)	The traveling salesman problem.
(xxi)	The graph isomorphism problem.
(xxii)	The 3-coloring problem.
(xxiii)	The 2-coloring problem.

(xxiv) _____ The set of binary numerals for Busy Beaver numbers.

7. Prove that every context-sensitive language is decidable. The way to do this is to start with an arbitrary non-contracting grammar, and then design a program which decides whether any given string is generated by that grammar. (If the string has length n, the running time of your program could be very long, maybe an exponentially bounded function of n.) You do not have to actually write the program, not even pseudocode; just explain how you would do it.

two-dimensional pieces, can they be

- 8. Prove that every decidable language is enumerated in canonical order by some machine.
- 9. Prove that every language that is enumerated in canonical order by some machine is decided by some other machine..
- 10. Prove that eny language accepted by any machine can be enumerated by some other machine.
- 11. Prove that any language which is enumerated by some machine is accepted by some other machine.
- 12. Prove that the halting problem is undecidable.
- 13. Prove that the grammar given in Problem 5 is ambiguous by giving two different leftmost derivations for some string. (If you simply give two different parse trees, you have not answered the question.)
- 14. Use the pumping lemma to prove that the Dyck language is not regular.
- 15. Give a polynomial time reduction of 3-SAT to to the independent set problem.
- 16. Give a polynomial time reduction of the subset sum problem to the partition problem.

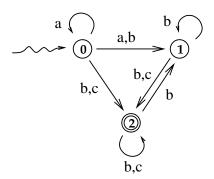
17. I have repeatedly stated in class that no language that has parentheses can be regular. For that to be true, there must be parenthetical strings of arbitrary nesting depth. (If you don't know what nesting depth is, look it up.) Some programming languages have limitations on nesting depth. For example, I have read that ABAP has maximum nesting depth of 256. (Who would ever want to go that far?)

The Dyck language is generated by the following context-free grammar. (As usual, to make grading easier, I use a and b for left and right parentheses.)

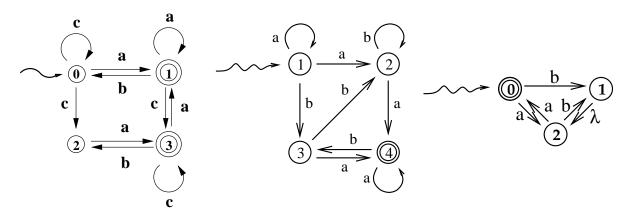
- 1. $S \rightarrow aSbS$
- 2. $S \rightarrow \lambda$

Prove that the subset of the Dyck language consisting of strings whose nesting depth is no more than 50 is regular.

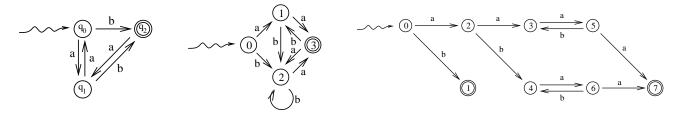
- 18. Find an NFA with at most 4 states which accepts the language of binary strings which contain the substring 111.
- 19. Let L be the language of all binary numerals for positive multiples of either 3 or 4, where leading zeros are not permitted. That is, $L = \{11, 100, 110, 1000, 1001, 1100, \ldots\}$. Find an NFA with 8 states which accepts L. (There is also a DFA with 12 states which accepts L.)
- 20. Construct a minimal DFA equivalent to the NFA shown below.



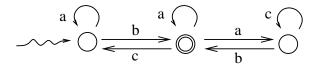
Find the minimal DFA equivalent to each of the following NFA.



21. Minimize each DFA:



- 22. Find an NFA which accepts the language generated by this grammar.
 - $S \rightarrow aA|cS|cC$
 - $A \rightarrow aA|bS|cB|\lambda$
 - $B \to aA|cB|bC|\lambda$
 - $C \to aB$
- 23. Give a regular expression which describes the language accepted by this NFA.



- 24. Find a DFA equivalent to the NFA shown in Figure 1.
- 25. Give a regular grammar for the language accepted by the machine in Figure 1.
- 26. Find a minimal DFA equivalent to the NFA shown in Figure 1.

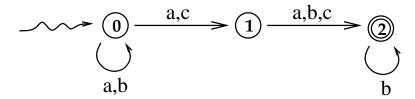


Figure 1: NFA for problems 26 and 27

- 27. Give a regular grammar with **no more than three variables** for the language accepted by the machine in Figure 1.
- 28. Give a regular expression for the language accepted by the machine in Figure ??
- 29. Illustrate an NFA which accepts the language generated by this grammar.
 - $S \to aA|cS|cC$
 - $A \to aA|bS|cB|\lambda$
 - $B \to aA|cB|bC|\lambda$
 - $C \to aB$

30. Let $L = \{w \in \{a, b\}^* : \#_a(w) = \#_b(w)\}$, which is generated by the following context-free grammar.

1.
$$S \rightarrow aSbS$$

2.
$$S \rightarrow bSaS$$

3.
$$S \rightarrow \lambda$$

Draw a DPDA which accepts L. (Recall that the input to that DPDA must be of the form w\$, where $w \in L$ and \$ is the end-of-file symbol.)

- 31. We know that context-free languages are exactly those which are accepted by push-down automata. We now define a new class of machines, which we call "limited push-down automata." An LPDA is exactly the same as a PDA, but with the restriction that the stack is never allowed to be larger than some given constant. What is the class of languages accepted by limited push-down automata? Think!
- 32. What is the class of languages decided by 2-PDA? A 2-PDA is the same as a PDA, except that it has two statcks instead of just one.
- 33. Use the CYK algorithm to decide whether abcab is generated by the CNF grammar. The start symbol is S.

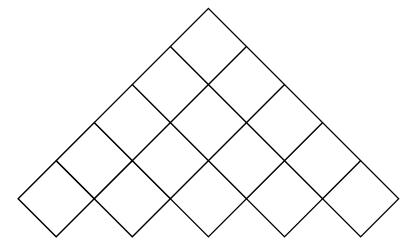
$$S \to AB \mid BC \mid CA$$

$$A \rightarrow a$$

$$B \rightarrow SA \mid SS \mid b$$

$$C \to c$$

by filling in the matrix.



34. Use the CYK algorithm to decide whether x - x - -x is generated by the CNF grammar below, by filling in the matrix. The start symbol is E.

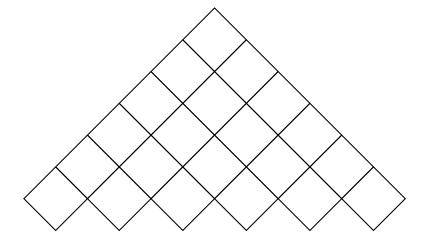
$$E \to ME$$

$$A \to EM$$

$$E \to AE$$

$$M \rightarrow -$$

$$E \to x$$



35. What complexity class contains all sliding block problems?

	(i)	The set of integers.
	(ii)	The set of rational numbers.
	(iii)	The set of real numbers.
	(iv)	The set of binary languages.
	(v)	The set of co- \mathcal{RE} binary languages.
	(vi)	The set of undecidable binary languages.
	(vii)	The set of functions from integers to integers.
((viii)	The set of recursive real numbers.
	(ix)	The set of $\mathcal{P} ext{SPACE}$ languages over the binary alphabet.
	(x)	The set of functions from the integers to the binary alphabet $\{0,1\}$.
	(xi)	The set of functions from the binary alphabet $\{0,1\}$ to the integers.
37.	Whie	ch class of languages does each of these machine classes accept?
	(i)	Deterministic finite automata.
	(ii)	Non-deterministic finite automata.
	(iii)	Push-down automata.
	(iv)	Turing Machines.
38.	parse	grammar below is an ambiguous CF grammar with start symbol E , and is parsed by the LALR er whose ACTION and GOTO tables are shown here. The ACTION table is missing actions for second column, when the next input symbol is the "minus" sign. Fill it in. Remember the C++

precedence of operators. (Hint: the column has seven different actions: s2, s4, r1, r2, r3, r4, and r5,

some more than once, and has no blank spaces.)

36. Label each of the following sets as countable or uncountable. Write ${\bf C}$ or ${\bf U}.$

1. $E \to E2 E_3$		\boldsymbol{x}	_	*	()	\$	S
$2. E \rightarrow4 E_5$	0	s11			s8			1
3. $E \rightarrow E *_6 E_7$	1			s6			halt	
4. $E \to (_8E_9)_{10}$ 5. $E \to x_{11}$	2	s11			<i>s</i> 8			3
	3			s6		r1	r1	
	4	s11			s8			5
	5			r2		r2	r2	
	6	s11			<i>s</i> 8			7

r3

s6

 $r4 \mid r4$

r5

39. What is the importance nowadays of \mathcal{NC} ?

7

8

9

10

11

s11

40. Consider the CF grammar below. The ACTION and GOTO tables are given below, except that six actions are mising, indicated by question marks. Fill in the missing actions (below the question marks). The actions of your table must be consistent with the precedence of operators in C++.

r3

s6

r4

r5

r3

r4

r5

9

		1	ı	1	1	1
		x	_	*	\$	$\mid E \mid$
1. $E \to E2 E_3$	0	s8	s4			1
$2. E \rightarrow4E_5$	1		<i>s</i> 2	s6	HALT	
3. $E \rightarrow E *_6 E_7$	2	s8	s4			3
4. $E \rightarrow x_8$	3		?	?	r1	
·						
	4	s8	s4			5
	5		?	?	r2	
	6	s8	s4			7
	7		?	?	r3	
	8	s8	r4	r4	r4	

41. When there is an "else" after two "if"s, which "if" does the "else" pair with? Here is CF grammar, G_3 , which isolates this problem. The start symbol S is the only variable. The symbol S is the only variable. The symbol S is "if(condition)", S represents "else," S represents "while(condition)" and S represents any other statement, such as an assignment statement.

	1	ACTION						
		ACTION						
	a	i	e	w	\$	S		
0	s8	s2		s6		1		
1					HALT			
2	s8	s2		s6		3		
3			s4		r1			
4	s8	s2		s6		5		
5			r2		r2			
6	s8	s2		s6		7		
7			r3		r3			
8			r4		r4			

- 1. $S \rightarrow i_2 S_3$
- 2. $S \rightarrow i_2 S_3 e_4 S_5$
- 3. $S \rightarrow w_6 S_7$
- 4. $S \rightarrow a_8$
- 8. Which entry, or entries, solve the dangling else problem?
- 9. Walk through the actions of the LALR parser for the input string iwiaewa.

stack	input	output	action
\$0	iwiaewa\$		

- 42. (i) Give a context-sensitive grammar for $\{a^nb^nc^n: n>0\}$
 - (ii) Using that grammar, give a derivation of the string aaabbbccc.
- 43. Work Exercise 1 on the handout reglrNC.pdf.