True/False Questions

$\mathcal{P}$ means $\mathcal{P}$–TIME
$\mathcal{NP}$ means $\mathcal{NP}$–TIME
$\mathcal{RE}$ means recursively enumerable
$\mathcal{NC}$ means Nick's class.
If $\mathcal{C}$ is any class of languages, co-$\mathcal{C}$ means the class of all languages which are complements of languages in $\mathcal{C}$.
A *binary language* is a language over the binary alphabet $\{0, 1\}$.
A *recursive* function is any function which can be computed by a machine.
A *recursive* real number is any real number whose n^{th} decimal digit is a recursive function of n.
A *fraction* is a string consisting of a numeral, followed by a slash, followed by another numeral.

1. True or False. T = true, F = false, and O = open, meaning that the answer is not known science at this time. In the questions below, $\mathcal{P}$ and $\mathcal{NP}$ denote $\mathcal{P}$-TIME and $\mathcal{NP}$-TIME, respectively.

   (i) ________ The problem of whether a given string is generated by a given context-free grammar is decidable.

   (ii) ________ If G is a context-free grammar, the question of whether $L(G) = \emptyset$ is decidable.

   (iii) ________ The Kleene closure of any $\mathcal{NP}$ langauge is $\mathcal{NP}$

   (iv) ________ The language $\{a^n b^n c^n d^n \mid n \geq 0\}$ is recursive.

   (v) ________ The language $\{a^n b^n c^n \mid n \geq 0\}$ is in the class $\mathcal{P}$-TIME.

   (vi) ________ There exists a polynomial time algorithm which finds the factors of any positive integer, where the input is given as a binary numeral.

   (vii) ________ Every undecidable problem is $\mathcal{NP}$-complete.

   (viii) ________ Every problem that can be mathematically defined has an algorithmic solution.

   (ix) ________ The intersection of two undecidable languages is always undecidable.

   (x) ________ Every $\mathcal{NP}$ language is decidable.

   (xi) ________ If L_1 and L_2 are $\mathcal{NP}$-complete languages and $L_1 \cap L_2$ is not empty, then $L_1 \cap L_2$ must be $\mathcal{NP}$-complete.

   (xii) ________ There exists a $\mathcal{P}$-TIME algorithm which finds a maximum independent set in any graph G.

   (xiii) ________ There exists a $\mathcal{P}$-TIME algorithm which finds a maximum independent set in any acyclic graph G.

   (xiv) ________ $\mathcal{NC} = \mathcal{P}$.

   (xv) ________ $\mathcal{P} = \mathcal{NP}$.

   (xvi) ________ $\mathcal{NP} = \mathcal{P}$-SPACE

(xvii) ________ $\mathcal{P}$-SPACE = EXP-TIME

(xviii) ________ EXP-TIME = EXP-SPACE

(xix) ________ The traveling salesman problem (TSP) is known to be $\mathcal{NP}$-complete.

(xx) ________ The language consisting of all satisfiable Boolean expressions is known to be $\mathcal{NP}$-complete.

(xxi) ________ The Boolean Circuit Problem is in $\mathcal{P}$.

(xxii) ________ The Boolean Circuit Problem is in $\mathcal{NC}$.

(xxiii) ________ 2-SAT is $\mathcal{P}$-TIME.

(xxiv) ________ 3-SAT is $\mathcal{P}$-TIME.

(xxv) ________ Primality is $\mathcal{P}$-TIME.

(xxvi) ________ There is a $\mathcal{P}$-TIME reduction of the halting problem to 3-SAT.

(xxvii) ________ Every context-free language is in $\mathcal{NC}$.

(xxviii) ________ Addition of binary numerals is in $\mathcal{NC}$.

(xxix) ________ Every language generated by a general grammar is recursive.

(xxx) ________ The problem of whether two given context-free grammars generate the same language is decidable.

(xxxi) ________ The language of all fractions (using base 10 numeration) whose values are less than π is decidable.

(xxxii) ________ For any two languages L_1 and L_2, if L_1 is undecidable and there is a recursive reduction of L_1 to L_2, then L_2 must be undecidable.

(xxxiii) ________ For any two languages L_1 and L_2, if L_2 is undecidable and there is a recursive reduction of L_1 to L_2, then L_1 must be undecidable.

(xxxiv) ________ If P is a mathematical proposition that can be written using a string of length n, and P has a proof, then P must have a proof whose length is $O(2^{2^n})$.

(xxxv) ________ If L is any $\mathcal{NP}$ language, there must be a $\mathcal{P}$–TIME reduction of L to the partition problem.

(xxxvi) ________ If L is $\mathcal{NP}$ and also co-$\mathcal{NP}$, then L must be $\mathcal{P}$.

(xxxvii) ________ A language is $\mathcal{RE}$ if and only if it is generated by a grammar.

(xxxviii) ________ If L is $\mathcal{RE}$ and also co-$\mathcal{RE}$, then L must be decidable.

(xxxix) ________ Every language is enumerable.

(xl) ________ If a language L is undecidable, then there can be no machine that enumerates L.

(xli) ________ There exists a mathematical proposition which is true, but can be neither proved nor disproved.

(xlii) ________ There is a non-recursive function which grows faster than any recursive function.

(xliii) ________ There exists a machine that runs forever and outputs the string of decimal digits of π (the well-known ratio of the circumference of a circle to its diameter).

(xliv) ________ For every real number x, there exists a machine that runs forever and outputs the string of decimal digits of x.

(xlv) ________ Every subset of any enumerable set is enumerable.

(xlvi) ________ There is a polynomial time reduction of the subset sum problem to the binary numeral factorization problem.

(xlvii) ________ For any real number x, the set of fractions whose values are less than x is $\mathcal{RE}$.

(xlviii) ________ For any recursive real number x, the set of fractions whose values are less than x is recursive (i.e., decidable).

(xlix) ________ The membership problem for any CFL is in the class $\mathcal{P}$-TIME.

(l) ________ 2-SAT is known to be $\mathcal{NP}$-complete.

(li) ________ The complement of any $\mathcal{P}$-TIME language is $\mathcal{P}$-TIME.

(lii) ________ The complement of any $\mathcal{P}$–SPACE language is $\mathcal{P}$–SPACE.

(liii) ________ The complement of any decidable language is decidable.

(liv) ________ The complement of any undecidable language is undecidable.

(lv) ________ The complement of any $\mathcal{RE}$ language is $\mathcal{RE}$.

The *jigsaw puzzle problem* is, given a set of various polygons, and given a rectangular table, is it possibe to assemble those polygons to exactly cover the table?

The *furniture mover's problem* is, given a room with a door, and given a set of objects outside the room, it is possible to move all the objects into the room through the door?

(lvi) ________ The jigsaw puzzle problem is known to be $\mathcal{NP}$ complete.

(lvii) ________ The jigsaw puzzle problem is known to be $\mathcal{P}$–SPACE complete.

(lviii) ________ The furniture mover's problem is known to be $\mathcal{NP}$ complete.

(lix) ________ The furniture mover's problem is known to be $\mathcal{P}$–SPACE complete.

(lx) ________ The complement of any recursive language is recursive.

(lxi) ________ For any infinite countable sets A and B, there is a 1-1 correspondence between A and B.

(lxii) ________ The set of all binary languages is countable.

(lxiii) ________ A language L is recursively enumerable if and only if there is a machine which accepts L.

(lxiv) ________ Every $\mathcal{NP}$ language is reducible to the independent set problem in polynomial time.

(lxv) ________ If a Boolean expression is satisfiable, there is a polynomial time proof that it is satisfiable.

(lxvi) ________ The general sliding block problem is $\mathcal{P}$–SPACE complete.

(lxvii) ________The halting problem is decidable.