CSC 456/656 Fall 2025 Answers to First Examination February 12, 2025

Name:____

The entire test is 245 points.

In the questions of this test, $\mathcal{P}$ and $\mathcal{NP}$ denote $\mathcal{P}$-TIME and $\mathcal{NP}$-TIME, respectively.
If L is a language over an alphabet Σ, we define the *complement* of L to be the set of all strings over Σ which are not in L. If $\mathcal{C}$ is a class of languages, we define co-$\mathcal{C}$ to be the class of all complements of members of $\mathcal{C}$.

1. True or False. 5 points each. T = true, F = false, and O = open, meaning that the answer is not known science at this time.

   (i) **F** Every subset of a regular language is regular.

   (ii) **T** The set of binary numerals for multiples of 23 is regular.

   (iii) **T** The set of binary numerals for prime numbers is in $\mathcal{P}$–TIME.

   (iv) **T** Every language is countable.

   (v) **F** The set of languages over the binary alphabet is countable.

   (vi) **O** $\mathcal{P} = \mathcal{NP}$.

   (vii) **T** The complement of any $\mathcal{P}$–TIME language is $\mathcal{P}$–TIME.

   (viii) **O** The complement of any $\mathcal{NP}$ language is $\mathcal{NP}$.

   (ix) **T** Every finite language is regular.

   (x) **T** A language is regular if and only if it is accepted by some DFA.

   (xi) **T** A language is regular if and only if it is accepted by some NFA.

   (xii) **T** A language is regular if and only if it is generated by some regular grammar.

   (xiii) **F** The programming language C++ is regular.

   (xiv) **T** The union of any two regular languages is regular.

   (xv) **T** The intersection of any two regular languages is regular.

   (xvi) **T** The concatenation of any two regular languages is regular.

   (xvii) **T** The Kleene closure of any regular language is regular.

   (xix) **O** There exists a polynomial time algorithm which finds the factors of any positive integer, where the input is given as a binary numeral.

(xx) **F** Every problem that can be mathematically defined has an algorithmic solution.

(xxi) **T** The intersection of any two $\mathcal{NP}$ languages is $\mathcal{NP}$.

(xxii) **O** $\mathcal{NP} = \mathcal{P}$-SPACE

2. [10 points] Suppose L is a problem such that you can check any suggested solution in polynomial time. Which one of these statements is certainly true?

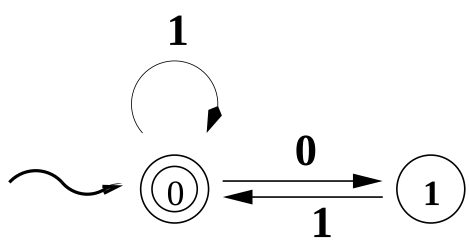
   (i) L is $\mathcal{P}$.

   (ii) L is $\mathcal{NP}$.

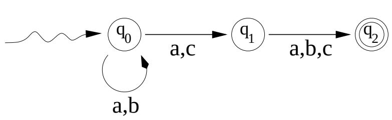
   (iii) L is $\mathcal{NP}$-complete.

   L is $\mathcal{NP}$.

3. [20 points] L be the language of all binary strings in which each 0 is followed by 1. Draw a DFA which accepts L.



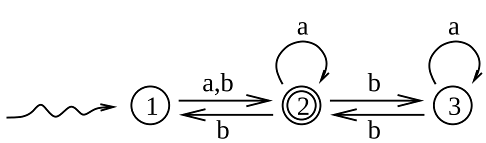
4. [20 points] Give a grammar, with at most 3 variables, for the language accepted by the follwing NFA.



Here are two different answers.

$S \to aS$	$S \to aS$
$S \to bS$	$S \to bS$
$S \to aA$	$S \to aA$
$S \to cA$	$S \to cA$
$A \to aB$	$A \to a$
$A \to bB$	$A \to b$
$A \to cB$	$A \to c$
$B \to \lambda$	

5. [20 points] Give a regular expression for the language accepted by the following NFA



   There are many (actually infinitely many) correct answers. But the simplest one, in my opinion, is $(a+b)(b(a+b)+a+ba^*a)^*$.

6. [10 points] Give an example of a language which is not regular. There are many. Two that have been mentioned in class are the Dyck language and $\{a^n b^n \ : \ n \geq 0\}$ "English" is not an acceptable answer, since we are not studying natural language.

7. [20 points] Give a definition of the class $\mathcal{P}$–TIME.

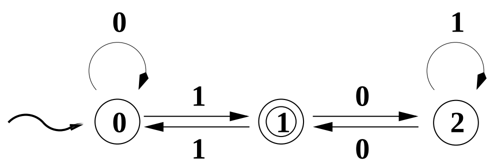
This question is phrased ambiguously. I meant the language class $\mathcal{P}$–TIME, but you might have interpreted it in a different way. I took that ambiguity into account while grading.

(i) A function $f : \mathcal{N} \to \mathcal{N}$ is in class $\mathcal{P}$ if there is some integer k such that $f(n) = O(n^k)$.

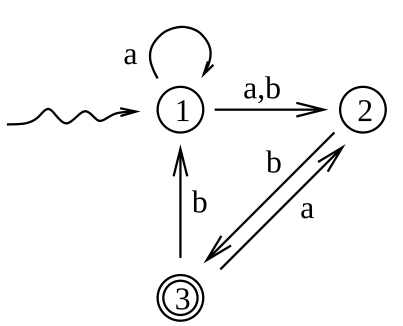
(ii) A problem P is in class $\mathcal{P}$–TIME if there is some $\mathcal{P}$ function f such that any instance of P described by n bits can be solved in $f(n)$ time by a deterministic machine (such as a computer program.)

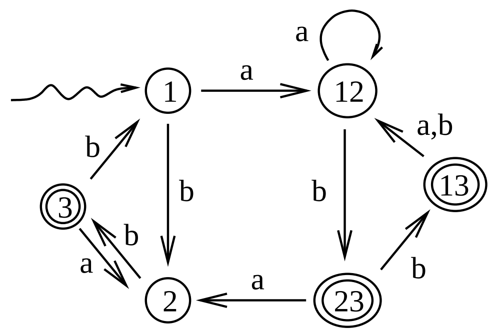
(iii) A language L is in class $\mathcal{P}$–TIME if the membership problem for L is in $\mathcal{P}$–TIME.

8. [20 points] Let L be the language of all binary strings encoding numbers which are equivalent to 1 modulo 3, where leading zeros are allowed. Thus, $L = \{1, 01, 001, 100, 111, 0100, 0111, 1010, \ldots\}$. Draw a DFA which accepts L. (You need only three states.)



9. [20 points] Draw a minimal DFA equivalent to the NFA shown in the figure below. Show the transition table, and also show the matrix used for minimizing the DFA.





	a	b
1	12	2
2	$\emptyset$	3
3	a	1
12	12	23
13	12	12
23	2	13

	1	2	3	12	13	23
1	O	X	X	X	X	X
2	X	O	X	X	X	X
3	X	X	O	X	X	X
12	X	X	X	O	X	X
13	X	X	X	X	O	X
23	X	X	X	X	X	O