University of Nevada, Las Vegas Computer Science 456/656 Spring 2026

Answers to Assignment 4: Due Tuesday March 31, 2026

1. State the pumping lemma for regular languages.

   For any regular language L
   There exists a positive number p such that
   For any string $w \in L$ of length at least p
   There exist strings x, y, z such that the following four statements hold
   1. $w = xyz$
   2. $|xy| \leq p$
   3. y is not the empty string
   4. For any integer $i \geq 0$ $xy^iz \in L$.

2. Use the pumping lemma for regular languages to prove that $L = \{a^nb^na^n : n \geq 0\}$ is not regular.

   The proof is by contradiction. Assume that L is regular. Let p be the pumping length of L. Let $w = a^pb^pc^p$, which is a member of L, and which has length at least p. Then there must exists strings x, y, z which satisfy the four conditions given by the pumping lemma. By the first condition, $a^pb^pa^p = xyz$, and by the second condition $|xy| \leq p$, thus $y = a^k$ for some k. by the third condition, $k \geq 1$. By the fourth condition, we can choose $i = 0$, and then $w' = xy^0z \in L$. But $\#_a(w') = 2p - k$ and $\#_b(w') = p$, contradiction, since $k > 0$.

3. Prove that every decidable language is enumerated in canonical order by some machine.

   Let L be a decidable language over an alphabet Σ. Let $w_1, w_2, \ldots$ be the canonical enumeration of Σ^*. The following program enumerates L in canonical order.

   For $i = 1$ to ∞
       If $w_i \in L$
           Write w_i

4. Prove that if a language L is enumerated in canonical order by some machine, L is decidable.

   Let $w_1, w_2, \ldots$ be the canonical enumeration of L. The following program can be executed since that enumeration is written by some machine. The program decides whether a given string w is in L.

   For $i = 1$ to ∞
       If $w = w_i$
           Halt and Accept
       Else if $w < w_i$ in the canonical ordering
           Halt and Reject

5. Prove that every language accepted by any machine is recursively enumerable.

   Let $L \subseteq \Sigma^*$ be a language accepted by a machine M. Let $w_1, w_2, \ldots$ be the canonical enumeration of Σ^*. The following program enumerates L.

   For $t = 1$ to ∞
       For $i = 1$ to t

If M accepts w_i within t steps
Write w_i

Note that each member of L will be written infinitely many times.

6. Prove that every recursively enumerable language is accepted by some machine.

   Let L be a recursively enumerable language, and let $w_1, w_2 \ldots$ be an enumeration of L computed by some machine. The following program accepts a string w if and only if $w \in L$.

   For $i = 1$ to ∞
   If $w = w_i$
   Halt and Accept

7. Finish the LALR parser for the following grammar, where E is the start symbol, and the language has both subtraction and negation. I have filled in all but one column.

   1. $E \to E -_2 E_3$
   2. $E \to -_4 E_5$
   3. $E \to (_6 E_7)_8$
   4. $E \to x_9$

	x	$-$	(	)	\$	E
0	$s9$	$s4$	$s6$			1
1		$s2$			halt	
2	$s9$	$s4$	$s6$			3
3		$r1$		$r1$	$r1$	
4	$s9$	$s4$	$s6$			5
5		$r2$		$r2$	$r2$	
6	$s9$	$s4$	$s6$			7
7		$s2$		$s8$		
8		$r3$		$r3$	$r3$	
9		$r4$		$r4$	$r4$	

8. Consider the annotated CF grammar and LALR parser. Walk through the computation of the parser for the input string $x + x + x * x * x \wedge x \wedge x$.

   1. $E \to E +_2 E_3$
   2. $E \to E *_4 E_5$
   3. $E \to E \wedge_6 E_7$
   4. $E \to x_8$

	x	$+$	$*$	$\wedge$	\$	E
0	$s8$					1
1		$s2$	$s4$	$s6$	halt	
2	$s8$					3
3		$r1$	$s4$	$s6$	$r1$	
4	$s8$					5
5		$r2$	$r2$	$s6$	$r2$	
6	$s8$					7
7		$r3$	$r3$	$s6$	$r3$	
8		$r4$	$r4$	$r4$	$r4$	

STACK	INPUT	OUTPUT	ACTION
$\$_0$	$x + x + x * x * x \wedge x \wedge x\$$		
$\$_0 x_8$	$+x + x * x * x \wedge x \wedge x\$$		$s8$
$\$_0 E_1$	$+x + x * x * x \wedge x \wedge x\$$	4	$r4$
$\$_0 E_1 +_2$	$x + x * x * x \wedge x \wedge x\$$	4	$s2$
$\$_0 E_1 +_2 x_8$	$+x * x * x \wedge x \wedge x\$$	4	$s8$
$\$_0 E_1 +_2 E_3$	$+x * x * x \wedge x \wedge x\$$	44	$r4$
$\$_0 E_1$	$+x * x * x \wedge x \wedge x\$$	441	$r1$
$\$_0 E_1 +_2$	$x * x * x \wedge x \wedge x\$$	441	$s2$
$\$_0 E_1 +_2 x_8$	$*x * x \wedge x \wedge x\$$	441	$s8$
$\$_0 E_1 +_2 E_3$	$*x * x \wedge x \wedge x\$$	4413	$r4$
$\$_0 E_1 +_2 E_3 *_4$	$x * x \wedge x \wedge x\$$	4414	$s4$
$\$_0 E_1 +_2 E_3 *_4 x_8$	$*x \wedge x \wedge x\$$	4414	$s8$
$\$_0 E_1 +_2 E_3 *_4 E_5$	$*x \wedge x \wedge x\$$	44143	$r4$
$\$_0 E_1 +_2 E_3$	$*x \wedge x \wedge x\$$	441442	$r2$
$\$_0 E_1 +_2 E_3 *_4$	$x \wedge x \wedge x\$$	441442	$s4$
$\$_0 E_1 +_2 E_3 *_4 x_8$	$\wedge x \wedge x\$$	441442	$s8$
$\$_0 E_1 +_2 E_3 *_4 E_5$	$\wedge x \wedge x\$$	441442	$r4$
$\$_0 E_1 +_2 E_3 *_4 E_5 \wedge_6$	$x \wedge x\$$	441442	$s6$
$\$_0 E_1 +_2 E_3 *_4 E_5 \wedge_6 x_8$	$\wedge x\$$	441442	$s8$
$\$_0 E_1 +_2 E_3 *_4 E_5 \wedge_6 E_7$	$\wedge x\$$	4414424	$r4$
$\$_0 E_1 +_2 E_3 *_4 E_5 \wedge_6 E_7 \wedge_6$	$x\$$	4414424	$s6$
$\$_0 E_1 +_2 E_3 *_4 E_5 \wedge_6 E_7 \wedge_6 x_8$	$\$$	4414424	$s8$
$\$_0 E_1 +_2 E_3 *_4 E_5 \wedge_6 E_7 \wedge_6 E_7$	$\$$	44144244	$r4$
$\$_0 E_1 +_2 E_3 *_4 E_5 \wedge_6 E_7$	$\$$	441442443	$r3$
$\$_0 E_1 +_2 E_3 *_4 E_5$	$\$$	4414424433	$r3$
$\$_0 E_1 +_2 E_3$	$\$$	44144244332	$r2$
$\$_0 E_1$	$\$$	441442443321	$r1$

HALT