

Context Sensitive Grammars and Languages

Like other grammars, a context sensitive grammar consists of variables, terminals and productions.

1. There are two alphabets, Σ , the alphabet of terminals, and Γ , the alphabet of variables.
2. $\Sigma \cap \Gamma = \emptyset$.
3. One member of Γ is the *start variable*. We usually write S for the start variable.
4. A *production* is written $lhs \rightarrow rhs$, where both lhs and rhs are strings over $\Sigma + \Gamma$, where $|lhs| \leq |rhs|$.
5. At least one symbol of lhs must be a variable.

Example

Let $L = \{a^n b^n c^n :: n > 0\}$. L is context-sensitive, but not context-free.

Context-sensitive
grammar for L

- | | |
|---------------------------|---|
| 1. $S \rightarrow abc$ | |
| 2. $S \rightarrow aaA bc$ | Context-Sensitive Derivation of $aaabbccc$: |
| 3. $A \rightarrow AaA$ | $S \Rightarrow aaA bc \Rightarrow aaAaA bc \Rightarrow aaAabbBc$ |
| 4. $Aa \rightarrow aA$ | $\Rightarrow aaAabbcc \Rightarrow aaaAbbcc \Rightarrow aaabbBbcc$ |
| 5. $Ab \rightarrow bbB$ | $\Rightarrow aaabbbBcc \Rightarrow aaabbccc$ |
| 6. $Bb \rightarrow bB$ | |
| 7. $Bc \rightarrow cc$ | |

Exercise

Find a context-sensitive grammar for the language L of all strings over the unary alphabet $\{1\}$ whose length is a power of 2. That is, $L = \{1, 11, 1111, 11111111, \dots\}$