CSC 456/656 S26 Answers to Study Guide for First Examination

1. True or False. T = true, F = false, and O = open, meaning that the answer is not known to science at this time.

   (i) **F** Every subset of a regular language is regular.

   (ii) **T** The set of binary numerals for prime numbers is in $\mathcal{P}$–TIME.

   (iii) **T** Every language is countable.

   (iv) **F** The set of languages over the binary alphabet is countable.

   (v) **O** $\mathcal{P} = \mathcal{NP}$.

   (vi) **T** The complement of any $\mathcal{P}$–TIME language is $\mathcal{P}$–TIME.

   (vii) **O** The complement of any $\mathcal{NP}$ language is $\mathcal{NP}$.

   (viii) **T** The complement of any decidable language is decidable.

   (ix) **O** If L is both $\mathcal{NP}$ and co–$\mathcal{NP}$, then L must be $\mathcal{P}$–TIME.

   (x) **T** If L is both $\mathcal{RE}$ and co–$\mathcal{RE}$, then L must be decidable.

   (xi) **T** Every context-free language is $\mathcal{P}$–TIME.

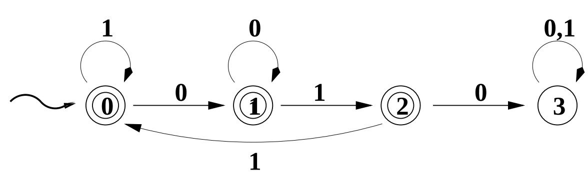
   (xii) **T** The halting problem is $\mathcal{RE}$.

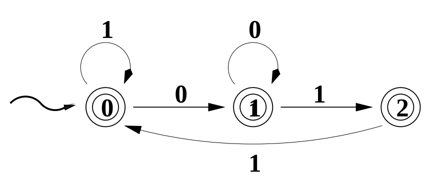
   (xiii) **F** The CFG equivalence problem is $\mathcal{RE}$.

   (xiv) **T** The language accepted by a DFA M contains the empty string if and only if the start symbol of M is final.

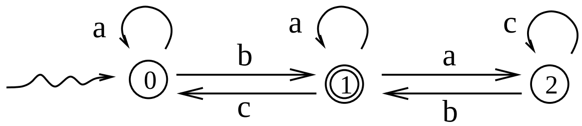
2. Let L be the language of all binary strings which do not contain the substring 010. Draw a DFA which accepts L.

   Here is the state diagram, with and without the dead state.

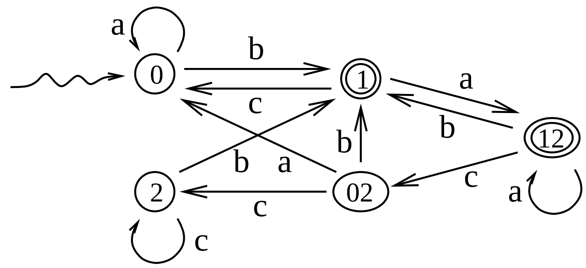




3. Construct a minimal DFA equivalent to M the NFA shown below.



	a	b	c
∅	∅	∅	∅
0	0	1	∅
1	12	∅	0
2	∅	1	2
12	12	1	02
02	0	1	2

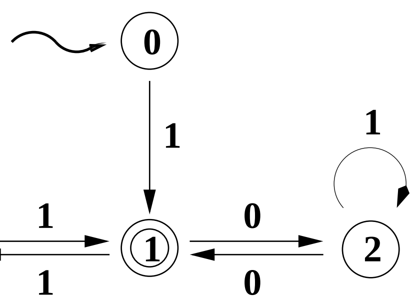


The DFA M' equivalent to M. To avoid clutter, the dead state of M' is not shown. By comparing pairs, it is quickly apparent that no two of its five live states are equivalent.

4. Give a regular expression for the language accepted by the NFA shown in problem 3. $a^*b(a+ca^*b+ac^*b)^*$

5. Give a language which is not regular. The simplest example is $L = \{a^nb^n\ :\ n \geq 0\}$

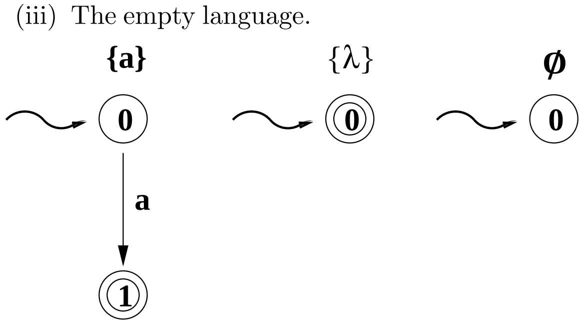
6. Let L be the set of binary numerals for positive integers which are equivalent to 1 modulo 3. ($\langle n \rangle$ such that $n\%3 = 1$) That is, $L = \{1, 100, 111, 1010, 1101, \ldots\}$. Draw a DFA that accepts L.



7. Draw a minimal DFA which accepts each of the following languages.
   (i) The language $\{a\}$.

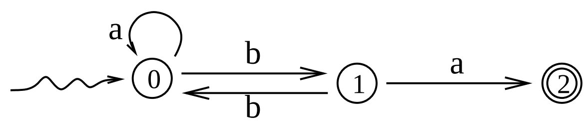
   (ii) The language $\{\lambda\}$.

   (iii) The empty language.



8. Draw the state diagram for a DFA which accepts the language generated by the following regular grammar. Hint: the DFA must have at least 3 states, not counting the dead state, which is not shown in the figure.

   1. $S \to aS$
   2. $S \to bA$
   3. $A \to a$
   4. $A \to bS$



9. If someone proved that every binary numeral could be factored into primes in polynomial time, what would be the practical consequence? Rivest Shamir Adelmann (RSA) encryptian could be broken in polynomial time.

10. State the *certificate/verification* definition of the class $\mathcal{NP}$.

A language L is $\mathcal{NP}$ if and only if there is a machine V_L and an integer k such that the following hold:

1. The input of V_L is (w, c) for strings w and c, and V halts in $O(|w|^k)$ time in either an accepting or a rejecting state.
2. If $w \in L$ there is a string c such that V_L accepts (w, c).
3. If $w \notin L$, then V does not accept (w, c) for any string c.