CS 456/656 Spring 2026 Examination March 11, 2026

No books, notes, scratch paper, or calculators. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided. If you want a scratch page to be graded, write "See scratch paper," on the test, and write your name on the scratch paper.

The entire examination is 325 points.

Name_____

1. True or False. T = true, F = false, and O = open, meaning that the answer is not known to science at this time. (5 points each)

   (i) _____ There is a unique minimal NFA which accepts any given regular language.

   (ii) _____ $\mathcal{P} \subseteq \mathcal{NP}$

   (iii) _____ Every subset of a regular language is regular.

   (iv) _____ If a language L can be defined mathematically, L must be decidable.

   (v) _____ $\mathcal{P}$–TIME = $\mathcal{P}$–SPACE

   (vi) _____ Every context-free language is accepted in polynomial time by some deterministic machine.

   (vii) _____ Every decidable language is recursively enumerable.

   (viii) _____ 2-SAT is $\mathcal{P}$–TIME.

   (ix) _____ 4-SAT is $\mathcal{P}$–TIME.

   (x) _____ $\mathcal{NP} \subseteq \mathcal{P}$–SPACE.

   (xi) _____ Every recursively enumerable language is decidable..

   (xii) _____ The class of regular languages is closed under Kleene closure.

   (xiii) _____ π^π is a recursive real number.

   (xiv) _____ The class of context-free languages is closed under union.

   (xv) _____ If L is $\mathcal{RE}$ and also co-$\mathcal{RE}$, then L must be decidable.

(xvi) _____ Any language accepted by any deterministic machine must be decidable.

(xvii) _____ The class of context-free languages is closed under intersection.

(xviii) _____ The set of binary numerals for prime numbers is a regular language.

(xix) _____ The Kleene closure of the empty language is empty.

(xx) _____ The complement of any $\mathcal{P}$-TIME language is $\mathcal{P}$-TIME.

(xxi) _____ The complement of any context-free language is context-free.

(xxii) _____ The complement of any recursive (that is, decidable) language is recursive.

(xxiii) _____ If Σ is an alphabet, then Σ^* is a regular language.

(xxiv) _____ If L is a language and L^* is a regular language, then L must be a regular language.

(xxv) _____ In regular expressions, concatenation distributes over union.

(xxvi) _____ The regular grammar equivalence problem is decidable.

(xxvii) _____ The context-free grammar equivalence problem is decidable.

(xxviii) _____ The regular expression equivalence problem is decidable.

(xxix) _____ The language of palindromes over $\{a, b\}$ is not accepted by any PDA.

(xxx) _____ Every context-free language is generated by an unambiguous context-free grammar.

(xxxi) _____ The Dyck language is context-free.

(xxxii) _____ The complement of $L = \{a^n b^n c^n \ : \ n \geq 0\}$ is context-free.

(xxxiii) _____ Every language is accepted by some machine.

(xxxiv) _____ If there is a computer program that decides whether a given string is a member of a language L, then L must be regular.

(xxxv) _____ If S is an infinite set, then S must have uncountably many subsets.

(xxxvi) _____ The definition of PDA allows for the possibility of multiple stacks.

(xxxvii) _____ If $w \in L$ and L is _____, there is a polynomial time proof that $w \in L$.

(xxxviii) _____ Every context-free language is $\mathcal{P}$–TIME.

2. Fill in the blanks.

   (i) [10 points] A language is context-free if and only if it is accepted by some _____.

   (ii) [10 points] If L_1 is $\mathcal{NP}$ and L_2 is _____, there must be a polynomial time reduction of L_1 to L_2.

3. [20 points] State the pumping lemma for regular languages.

4. [20 points] What are the four language (or grammar) classes of the Chomsky hierarchy? Be sure to mention the type numbers as well as the names of the classes.

5. [20 points] Give a polynomial time reduction from 3-SAT to the Independent Set problem.

6. [20 points] Give a polynomial time reduction from the Subset Sum problem to the Partition problem.

7. [20 points] Finish the LALR parser for the following grammar, where E is the start symbol, and the language has both subtraction and negation. I have filled in all but one column.

1. $E \to E -_2 E_3$
2. $E \to -_4 E_5$
3. $E \to (_6 E_7)_8$
4. $E \to x_9$

	x	$-$	(	)	\$	E
0	$s9$		$s6$			1
1					halt	
2	$s9$		$s6$			3
3				$r1$	$r1$	
4	$s9$		$s6$			5
5				$r2$	$r2$	
6	$s9$		$s6$			7
7				$s8$		
8				$r3$	$r3$	
9				$r4$	$r4$	

8. [20 points] Below there is an annotated CFgrammar and an LALR parser. Walk through the computation of the parser for the input string $x + x + x * x$.

1. $E \to E +_2 E_3$
2. $E \to E *_4 E_5$
3. $E \to x_6$

	x	$+$	$*$	\$	E
0	$s6$				1
1		$s2$	$s4$	halt	
2	$s6$				3
3		$r1$	$s4$	$r1$	
4	$s6$				5
5		$r2$	$r2$	$r2$	
6		$r3$	$r3$	$r3$	