

# CS 456/656 Answers to Spring 2026 Examination February 11, 2026

1. True or False. T = true, F = false, and O = open, meaning that the answer is not known to science at this time. (5 points each)
  - (i) **T** There is a unique minimal DFA which accepts any given regular language.
  - (ii) **F** There is a unique minimal NFA which accepts any given regular language.
  - (iii) **F** Every context-free language is accepted by some deterministic PDA.
  - (iv) **O**  $\mathcal{P} = \mathcal{NP}$
  - (v) **F** Every subset of a regular language is regular.
  - (vi) **F** If a language  $L$  can be defined mathematically,  $L$  must be decidable.
  - (vii) **O**  $\mathcal{P}\text{-TIME} = \mathcal{P}\text{-SPACE}$
  - (viii) **T** Every context-free language is accepted in polynomial time by some deterministic machine.
  - (ix) **T** The class of regular languages is closed under Kleene closure.
  - (x) **T** The class of context-free languages is closed under union.
  - (xi) **T** If  $L$  is  $\mathcal{RE}$  and also  $\text{co-}\mathcal{RE}$ , then  $L$  must be decidable.
  - (xii) **F** Any language accepted by any deterministic machine must be decidable.
  - (xiii) **F** The class of context-free languages is closed under intersection.
  - (xiv) **F** The set of binary numerals for prime numbers is a regular language.
  - (xv) **F** The Kleene closure of the empty language is empty.
  - (xvi) **T** The complement of any  $\mathcal{P}\text{-TIME}$  language is  $\mathcal{P}\text{-TIME}$ .
  - (xvii) **F** The complement of any context-free language is context-free.
  - (xviii) **T** The complement of any recursive (that is, decidable) language is recursive.
  - (xix) **T** If  $\Sigma$  is an alphabet, then  $\Sigma^*$  is a regular language.

- (xx) **F** If  $L$  is a language and  $L^*$  is a regular language, then  $L$  must be a regular language.
  - (xxi) **T** In regular expressions, concatenation distributes over union.
  - (xxii) **T** The regular grammar equivalence problem is decidable.
  - (xxiii) **F** The context-free grammar equivalence problem is decidable.
  - (xxiv) **T** The regular expression equivalence problem is decidable.
  - (xxv) **F** The language of palindromes over  $\{a, b\}$  is not accepted by any PDA.
  - (xxvi) **F** Any context-free language is generated by some unambiguous context-free grammar.
  - (xxvii) **T** The Dyck language is context-free.
  - (xxviii) **T** The complement of  $L = \{a^n b^n c^n : n \geq 0\}$  is context-free. (We never did this in class, but if you think, you can figure it out using material we have covered.)
  - (xxix) **F** Every language is accepted by some machine.
  - (xxx) **F** If there is a computer program that decides whether a given string is a member of a language  $L$ , then  $L$  must be regular.
  - (xxxi) **F** A number  $N$  is called *uncountable* if no human being could count up to  $N$  in his or her lifetime.
  - (xxxii) **T** If  $S$  is an infinite set, then  $S$  must have uncountably many subsets.
  - (xxxiii) **F** The definition of PDA allows for the possibility of multiple stacks.
2. [10 points] A language is context-free if and only if it is accepted by some **PDA**.
3. [15 points] Given an unambiguous context-free grammar for the Dyck language, where (for ease of grading) you write “a” instead of “(” and “b” instead of “)” Partial credit for an ambiguous grammar.

I will give two answers, both of which are unambiguous.

1.  $S \rightarrow aSbS$

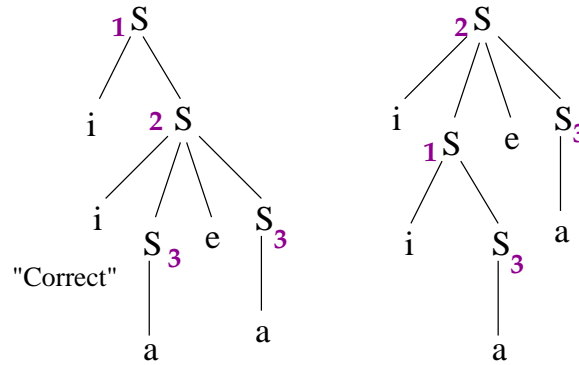
2.  $S \rightarrow \lambda$

1.  $S \rightarrow SaSb$

2.  $S \rightarrow \lambda$

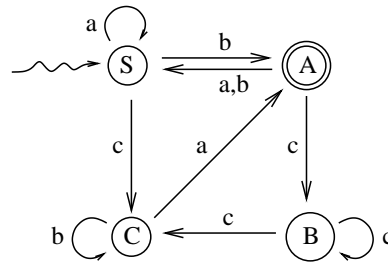
4. [20 points] Let  $G$  be the CF grammar given below, where  $S$  is the start symbol.  $G$  is ambiguous by giving two different parse trees for some string generated by  $G$ .

1.  $S \rightarrow iS$
2.  $S \rightarrow iSeS$
3.  $S \rightarrow wS$
4.  $S \rightarrow a$



5. [20 points] Draw an NFA which accepts the language generated by this grammar.

- $$S \rightarrow aS|bA|cC$$
- $$A \rightarrow aS|bS|cB|\lambda$$
- $$B \rightarrow cB|cC$$
- $$C \rightarrow aA|bC$$



6. [20 points] What are the four language (or grammar) classes of the Chomsky hierarchy? Be sure to mention the type numbers as well as the name of the class.

Type 3	Regular grammars	Regular languages
Type 2	Context-free grammars	Context-free languages
Type 1	Context-sensitive grammars	Context-sensitive languages
Type 0	Unrestricted-grammars	Recursively enumerable languages

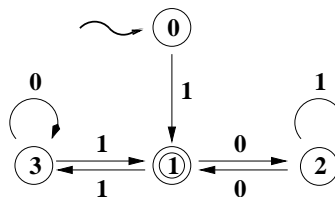
7. [20 points] Suppose someone publishes a correct algorithm that factors any positive integer in polynomial time. Name an important consequence of that discovery.

RSA encryption could be broken in polynomial time.

8. (a) [15 points] What language does the DFA below accept? (Hint: Arithmetic.)

It is the language of binary numerals for integers equivalent to 1 modulo 3.

- (b) [15 points] Write a regular expression for that language.  $1(01^*0 + 10^*1)^*$ .



9. [20 points] Find a minimal DFA equivalent to the NFA shown below.

