CS 456/656 Answers to Spring 2026 Examination March 11, 2026

No books, notes, scratch paper, or calculators. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided. If you want a scratch page to be graded, write "See scratch paper," on the test, and write your name on the scratch paper.

The entire examination is 320 points.

Name____

1. True or False. T = true, F = false, and O = open, meaning that the answer is not known to science at this time. (5 points each)

   (i) **F** There is a unique minimal NFA which accepts any given regular language.

   (ii) **T** $\mathcal{P} \subseteq \mathcal{NP}$

   (iii) **F** Every subset of a regular language is regular.

   (iv) **F** If a language L can be defined mathematically, L must be decidable.

   (v) **O** $\mathcal{P}$–TIME = $\mathcal{P}$–SPACE

   (vi) **T** Every context-free language is accepted in polynomial time by some deterministic machine.

   (vii) **T** Every decidable language is recursively enumerable.

   (viii) **T** 2-SAT is $\mathcal{P}$–TIME.

   (ix) **O** 4-SAT is $\mathcal{P}$–TIME.

   (x) **T** $\mathcal{NP} \subseteq \mathcal{P}$–SPACE.

   (xi) **F** Every recursively enumerable language is decidable..

   (xii) **T** The class of regular languages is closed under Kleene closure.

   (xiii) **T** π^π is a recursive real number.

   (xiv) **T** The class of context-free languages is closed under union.

   (xv) **T** If L is $\mathcal{RE}$ and also co-$\mathcal{RE}$, then L must be decidable.

(xvi) **F** Any language accepted by any deterministic machine must be decidable.

(xvii) **F** The class of context-free languages is closed under intersection.

(xviii) **F** The set of binary numerals for prime numbers is a regular language.

(xix) **F** The Kleene closure of the empty language is empty.

(xx) **T** The complement of any $\mathcal{P}$-TIME language is $\mathcal{P}$-TIME.

(xxi) **F** The complement of any context-free language is context-free.

(xxii) **T** The complement of any recursive (that is, decidable) language is recursive.

(xxiii) **T** If Σ is an alphabet, then Σ^* is a regular language.

(xxiv) **F** If L is a language and L^* is a regular language, then L must be a regular language.

(xxv) **T** In regular expressions, concatenation distributes over union.

(xxvi) **T** The regular grammar equivalence problem is decidable.

(xxvii) **F** The context-free grammar equivalence problem is decidable.

(xxviii) **T** The regular expression equivalence problem is decidable.

(xxix) **F** The language of palindromes over $\{a, b\}$ is not accepted by any PDA.

(xxx) **F** Every context-free language is generated by an unambiguous context-free grammar.

(xxxi) **T** The Dyck language is context-free.

(xxxii) **T** The complement of $L = \{a^n b^n c^n \,:\, n \geq 0\}$ is context-free.

(xxxiii) **F** Every language is accepted by some machine.

(xxxiv) **F** If there is a computer program that decides whether a given string is a member of a language L, then L must be regular.

(xxxv) **T** If S is an infinite set, then S must have uncountably many subsets.

(xxxvi) **F** The definition of PDA allows for the possibility of multiple stacks.

(xxxviii) **T** Every context-free language is $\mathcal{P}$–TIME.

2. Fill in the blanks.

   (i) [10 points] A language is context-free if and only if it is accepted by some **PDA**.

   (ii) [10 points] If L_1 is $\mathcal{NP}$ and L_2 is $\mathcal{NP}$–**complete**, there must be a polynomial time reduction of L_1 to L_2.

3. [20 points] State the pumping lemma for regular languages.

For any regular language L
There is a positive integer p, called the pumping length of L such that
For any string $w \in L$ of length at least p
There are strings x, y, and z such that the following statements hold

$xyz = w$
$|xy| \leq p$
y is not the empty string
For any integer $i \geq 0$ $xy^iz \in L$

4. [20 points] What are the four language (or grammar) classes of the Chomsky hierarchy? Be sure to mention the type numbers as well as the names of the types.

Regular languages are generated by regular, or type 3, grammars.

Context-free languages are generated by context-free, or type 2, grammars.

Context-sensitive languages are generated by context-sensitive, or type 1, grammars.

Recursively enumerable languages are generated by type 0, or unrestricted, or general, or phase-structure, grammars.

5. [20 points] Give a polynomial time reduction from 3-SAT to the Independent Set problem.

Let $e = C_1 * C_2 * \ldots C_K$ be an instance of 3-SAT, where C_i is the conjunction of three terms, $C_i = t_{i,1} + t_{i,2} + t_{i,3}$ ane each term is either a variable or the negation of a variable. The reduction maps e to a graph $G = (V, E)$ which has $3K$ vertices There is an edge from $v_{i,j}$ to $v_{i',j'}$ if and only if either $i = i'$ or $t_{i,j} * t_{i',j'}$ is a contradiction, meaning that one of those terms is the negation of the other. We need to prove that e has a satisfying assignment if and only if G has an independent set of vertices of order K.

Let α be a satisfying assignment of e. For each clause C_i of e we select one term of C_i which is true under that assignment, and let T be the set of those terms. Let V be the set of vertices of G corresonding to T. Then $|V| = |T| = K$. Two vertices $v_{i,j}, v_{i',j'} \in V$ cannot be connected by an edge of G, since $i \neq i'$, and $t_{i,j}$ and $t_{i',j'}$ do not contradict. Thus, V is an independent set of vertices of G of order K.

6. [20 points] Give a polynomial time reduction of the subset sum problem to partition.

An instance of the subset sum problem is an ordered pair (X, K) where K is a positive number, and $X = \{x_1, \ldots x_n\}$ is a set of positive numbers. That instance has a solution if the total of some subset of X is K.

An instance of the partiton problem is a set of positive numbers Y. That instance has a solution if Y can be partitioned into two subsets of equal weight.

We construct a mapping from each instance I of the subset sum problem to an instance J of the partition probem, such that J has a solution if and only if I has a solution.

Let $I = (X, K)$ an instance of subset sum. Let S be the sum of all members of X. We constuct an instance Y of partition. Let S be the sum of all members of X. Without loss of generality, $x_i \leq K$ for $x_i \in X$ and $K \leq S$. Let $Y = \{x_1, x_2, \ldots x_n, K + 1, S - K + 1\}$. The sum of the elements of Y is $2S + 2$.

Suppose I has the solution $A \subseteq X$. We letThe sum of the terms of B is $K + S - K + 1 = S + 1$, half the total of Y, hence B is a solution to the partition problem Y.

Conversely, suppose B is a a subset of Y whose total is $S + 1$. the terms $K + 1$ and $S - K + 1$ total more than $S + 1$, hence must be in opposite sides of the partition of Y. Without loss of generality, $S - K + 1 \in B$. The set obtained by removing $S - K + 1$ from B has total K, hence is a solution to I.

7. [20 points] Finish the LALR parser for the following grammar, where E is the start symbol, and the language has both subtraction and negation. I have filled in all but one column.

1. $E \to E -_2 E_3$
2. $E \to -_4 E_5$
3. $E \to (_6 E_7)_8$
4. $E \to x_9$

	x	$-$	(	)	\$	E
0	$s9$	$s4$	$s6$			1
1		$s2$			halt	
2	$s9$	$s4$	$s6$			3
3		$r1$		$r1$	$r1$	
4	$s9$	$s4$	$s6$			5
5		$r2$		$r2$	$r2$	
6	$s9$	$s4$	$s6$			7
7		$s2$		$s8$		
8		$r3$		$r3$	$r3$	
9		$r4$		$r4$	$r4$	

8. [20 points] Below there is an annotated CF-grammar and an LALR parser. Walk through the computation of the parser for the input string $x + x + x * x$.

1. $E \to E +_2 E_3$
2. $E \to E *_4 E_5$
3. $E \to x_6$

	x	$+$	$*$	\$	E
0	$s6$				1
1		$s2$	$s4$	halt	
2	$s6$				3
3		$r1$	$s4$	$r1$	
4	$s6$				5
5		$r2$	$r2$	$r2$	
6		$r3$	$r3$	$r3$	

STACK	INPUT	OUTPUT	ACTION
$\$_0$	$x + x + x * x\$$		
$\$_0 x_6$	$+x + x * x\$$		$s6$
$\$_0 E_1$	$+x + x * x\$$	3	$r3$
$\$_0 E_1 +_2$	$x + x * x\$$	3	$s2$
$\$_0 E_1 +_2 x_6$	$+x * x\$$	3	$s6$
$\$_0 E_1 +_2 E_3$	$+x * x\$$	33	$r3$
$\$_0 E_1$	$+x * x\$$	331	$r1$
$\$_0 E_1 +_2$	$x * x\$$	331	$s2$
$\$_0 E_1 +_2 x_6$	$*x\$$	331	$s6$
$\$_0 E_1 +_2 E_3$	$*x\$$	3313	$r3$
$\$_0 E_1 +_2 E_3 *_4$	$x\$$	3313	$s4$
$\$_0 E_1 +_2 E_3 *_4 x_6$	\$	3313	$s6$
$\$_0 E_1 +_2 E_3 *_4 E_5$	\$	33133	$r3$
$\$_0 E_1 +_2 E_3$	\$	331332	$r2$
$\$_0 E_1$	\$	3313321	$r1$

HALT