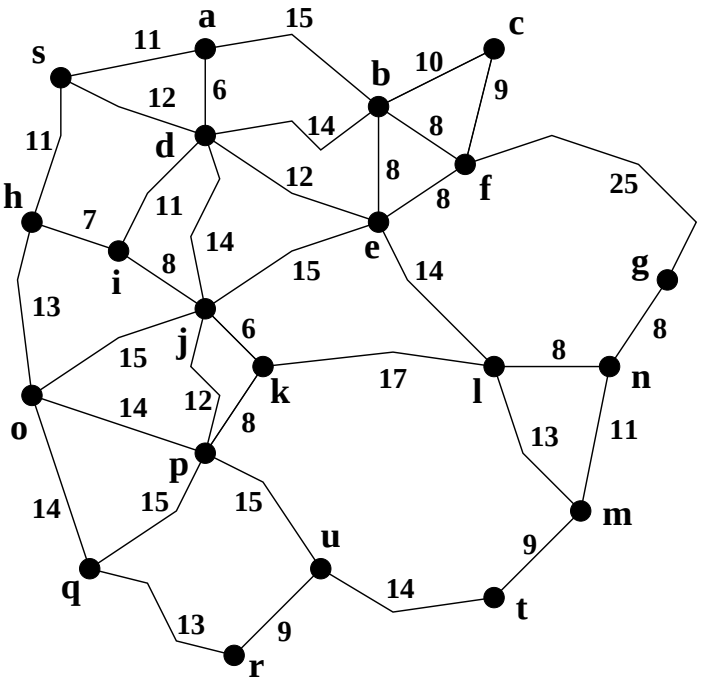
1. Work a problem from your homework.

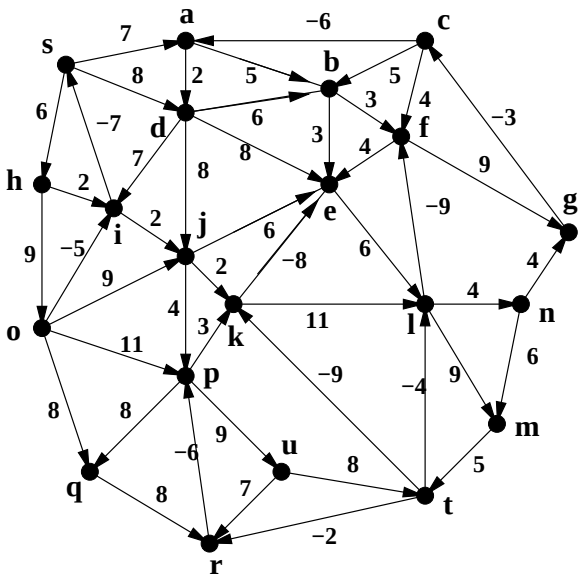
2. Explain how the A^* algorithm works. What is its time complexity? What is its memory complexity? Why is it "better"? Walk through the A^* algorithm to find the minimum distance from **s** to **t** for the following weighted graph, using the estimated distance given in the table.



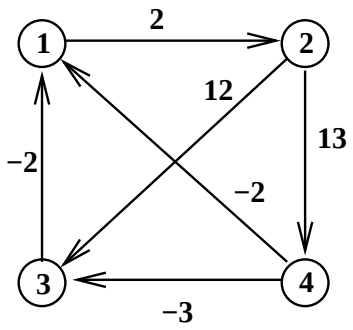
Estimated Distance to t

a	**52**
b	**38**
c	**38**
d	**50**
e	**30**
f	**31**
g	**28**
h	**43**
i	**51**
j	**30**
k	**24**
l	**16**
m	**9**
n	**20**
o	**43**
p	**25**
q	**36**
r	**20**
s	**59**
u	**14**

3. Explain how Johnson's algorithm works. What is its time complexity? What is its memory complexity? In the following example, sketch the augmented graph, and compute the results of the first stage of Johnson's algorithm. Then, use those results to reweight the original graph. Show the reweighted graph.



4. Explain how the Floyd-Warshall algorithm works. What is its time complexity? What is its memory complexity? Walk through the Floyd-Warshall algorithm for the following example:



5. (a) Does there exist a sorting algorithm that sorts four items using only five gates?
   (b) Draw a sorting network to sort five items which consists of fewer then ten gates.

6. Consider all finite sequences whose terms are positive integers no greater than 4. We say that such a sequence $x_1, x_2, \ldots x_n$ is *good* if the following hold:

   - The sum of any two consecutive terms is not five.
   - Two consecutive terms cannot both be 1.
   - For any $1 \leq i < n$, if $x_i = 4$, then $x_{i+1} \neq 3$.

   Let $F(n)$ = the number of good sequences of length n, and let $G(n) = F(n)$ modulo 31.[1] For example, $G(1) = 4$, $G(2) = 10$, $G(3) = 25$, and $G(5) = 30$.

   (a) Design an $O(n)$-time dynamic programming algorithm that computes $G(n)$.

   (b) Design an $O(\log n)$-time algorithm that computes $G(n)$. (Could be dynamic programming, could be something else, could be a mixture: who knows?)

[1] Putting in the **mod** operation eliminates the confusion between time complexity and number of arithmetic operations.