University of Nevada, Las Vegas Computer Science 477/677 Fall 2024

Assignment 1: Due Saturday August 30, 2025 23:59:59

Follow our TA, Louis DuMontet's (`dumontet@unlv.nevada.edu`) instructions on how to turn in the assignment.

Name:____

You are permitted to work in groups, get help from others, read books, and use the internet.

1. The sequence of powers of 2 is generated by the recurrence $2^n = 2 \cdot 2^{n-1}$. What is the recurrence which generates the Fibonacci sequence $F_1, F_2, \ldots$?

   $F_n = F_{n-1} + F_{n-2}$

2. Write the sequence of all Fibonacci numbers under 100.

   1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89

3. Which one of these statements is true?

   (a) The Fibonacci numbers increase logarithmically.

   (b) The Fibonacci numbers increase linearly.

   (c) The Fibonacci numbers increase quadratically.

   (d) The Fibonacci numbers increase exponentially.

   Exponentially, as we show in the next problem.

4. Find the constant K such that $F_n = \Theta(K^n)$. What is the standard name of this constant?

   The following argument looks like a proof, but it really isn't, since lots of details are missing.

   Assume that $F_n = K^n$, which is approximately true. Using the recurrence from problem 1, we have

$$\begin{aligned}
K^n &= K^{n-1} + K^{n-2} \\
&\quad \text{Dividing both sides by} K^{n-2}: \\
K^2 &= K + 1 \\
K^2 - K - 1 &= 0 \\
&\quad \text{Which is a quadratic equation. We obtain:} \\
K &= \frac{1 \pm \sqrt{5}}{2} \\
&\quad \text{But } K \text{ can't be negative, hence} \\
K &= \frac{1 + \sqrt{5}}{2}
\end{aligned}$$

   This constant is known as the "golden ratio."

In this course, I expect you to understand logarithms. For numbers $x > 0$ and $b > 1$, the base b logarithm of x is written $\log_b x$. The base is usually not written, but is understood to be the "default" base, which depends on the application. For any base, $\log(xy) = \log x + \log y$, $\log(x^y) = y \log x$, and $\log_b x = \log x / \log b$.

(i) In science and engineering, the default base is 10. That is if $\log x$ appears in a scientific discussion or manuscript, it means $\log_{10} x$.

(ii) In mathematics, the default base is $e \approx 2.718$, and $\log_e x$ is usually written $\ln x$.

(iii) In computer science, or computing in general, the default base is 2. Thus $\log 2 = 1$, $\log 4 = 2$, $\log 8 = 3$, and $\log 65536 = 16$. If I write $\log x$ in a homework assignment or an exam, I mean $\log_2 x$.

5. If you write `log(x)` in a C++ program, what is the base of the logarithm?

6. There are quantities that are normally expressed using a logarithmic scale. Describe four of them.

   (a) Ph to measure acidity.

   (b) The Richter scale for earthquakes.

   (c) Bel (usually, decibel is used.)

   (d) Magnitude of stars, such as apparent magnitude, absolute magnitude, bolometric magnitude.

   Hint: Chemistry, seismology, acoustics, astronomy.

7. You've seen Landau notation. Originally, there was only "big O," but now there are several others. We will only use three of those this semester.

   When we write $f(n) = O(g(n))$, we mean that there are constants C and N such that $f(n) \leq C\, g(n)$ for all $n \geq N$.

   When we write $f(n) = \Omega(n)$, we mean that there are constants C and N such that $f(n) \geq C\, g(n)$ for $n \geq N$.

   When we write $f(n) = \Theta(g(n))$ we mean that both $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$.

   Thus:

   $n^2 = O(n^3)$, and $n^3 = \Omega(n^2)$.

   $n^2 + 5n + 3 = \Theta(n^2)$.

8. Write either O, Ω or Θ in each blank. Write Θ if that is correct, otherwise write O or Ω. Think carefully. Some of these require thought.

   (a) $n - 100 = \Theta(n - 200)$

   (b) $n^{1/2} = O\left(n^{2/3}\right)$ since $1/2 < 2/3$.

   (c) $100n + \log n = \Theta\left(n + \log^2 n\right)$

   Hint: $\log n$ grows more slowly than any polynomially increasing function of n.

   (d) $n \log n = \Omega\left(10n + \log(10n)\right)$

(e) $\log(n^2) = \Theta\left(\log(n^3)\right)$

(f) $10 \log n = O\left(\log(n^2)\right)$

(g) $n^{1.01} = \Omega\left(n \log^2 n\right)$

(h) $n^2 / \log n = \Omega\left(n \log^2 n\right)$

This one requires serious thinking. Don't just write down the first thing that occurs to you.

(i) $n^{0.1} = \Omega\left(\log^2 n\right)$

The rest of these are harder, and may require calculation.

(j) $(\log n)^{\log n} = \Omega\left(n / \log n\right)$

(k) $\sqrt{n} = \Omega\left(\log^3 n\right)$

This one requires writing things down.

(l) $n^{1/2} = O\left(5^{\log_2 n}\right)$

Think!

(m) $n2^n = O\left(3^n\right)$

This one is (slightly) tricky.

(n) $2^n = \Theta\left(2^{n+1}\right)$

This one is easy, if you think about it correctly.

(o) $n! = \Omega\left(2^n\right)$

Don't forget that the default base is 2.

(p) $\log n^{\log_2 n} = O\left(2^{(\log n)^2}\right)$

If you know your calculus, this one is easy.

(q) $\sum_{i=1}^{n} i^k = \Theta\left(n^{k+1}\right)$

This next one is quite important for analyzing the time complexity of sorting algorithms, and it **will** appear on exams, and very likely during job interviews.

(r) $\log n! = \Theta\left(n \log n\right)$

9. Consider the following C++ program.

```
void process(int n)
 {
  cout << n << endl;
  if(n > 1) process(n/2);
  cout << n%2;
 }

int main()
 {
  int n;
  cout << "Enter a positive integer: ";
  cin >> n;
  assert(n > 0);
  process(n);
  cout << endl;
  return 1;
 }
```

The last line of the output of `process(n)` is a string of bits. What does this bitstring represent?

The binary numeral for n.

10. The C++ code below implements a function, "mystery." What does it compute?

```
float mystery(float x, int k)
 {
  if (k == 0) return 1.0;
  else if(x == 0.0) return 0.0;
  else if (k < 0) return 1/mystery(x,-k);
  else if (k%2) return x*mystery(x,k-1);
  else return mystery(x*x,k/2);
 }
```

x^k