University of Nevada, Las Vegas Computer Science 477/677 Fall 2025

Answers to Assignment 5: Due Thursday October 23, 2025

This assignment does not count for your grade, so don't turn it in. But be sure to work it!

I will insert additional problems later, but start on these right away.

Name:_____

You are permitted to work in groups, get help from others, read books, and use the internet.

1. True/False/Open

   (a) **F** Every mathematically defined problem can be solved by computation, although it might not have been done yet.

   (b) **F** "Many hands make light work." Any polynomial time problem can be worked in $O(\log^k n)$ time, for some k, by polynomially many processers working in parallel.

   (c) **F** Computers are so fast nowadays that analyzing algorithms for time complexity is pointless.

2. Fill in the blanks. In questions where the answer is a number, you must give an exact answer.

   (a) Bad question! Deleted.

   (b) Bad question! Deleted.

   (c) Bad question! Deleted.

   (d) **Dijkstra's** algorithm does not permit the weight of any arc to be negative.

3. Write pseudocode for the Bellman-Ford algorithm. Be sure to include the shortcut. Assume that the vertices are numbered from 0 to $n-1$ with source vertex 0, and that the arcs are numbered from 0 to $m-1$, and that the j^{th} arc is $a_j = (s_j, t_j)$ for vertices s_j and t_j, and has weight w_j. Be sure to compute back pointers.

```
V[0] = 0
For all i from 1 to n-1
    V[i] = infinity
bool finished = false
While(not finished)
    finished = true
    for j from 0 to m-1
        temp = V[t_j] + w_j
        If temp < V[s_j]
            s_j = temp
            back t_j = s_j
            finished = false
```

4. Solve the following recurrences. Express the answers using Θ notation.

(a) $F(n) = 3F(2n/3) + 3F(n/3) + n^3$

By Akra-Brazzi. $\Theta(n^3 \log n)$

(b) $F(n) = F(n/2) + 2F(n/4) + 1$

By Akra-Brazzi. $\Theta(n)$

(c) $F(n) = F(n/2) + F(n/3) + n$

By Akra-Brazzi. $\Theta(n)$

(d) $F(n) = 9F(n/3) + n$

By the Master Theorem. $\Theta(n^2)$.

(e) $F(n) = 3F(n/3) + n$;

By the Master Theorem. $\Theta(n \log n)$.

(f) $F(n) = F(n/3) + 1$;

By the Master Theorem. $\Theta(\log n)$.

(g) $F(n) = F(n-1) + \log n$;

By the anti-derivative method. $\Theta(n \log n)$.

(h) $F(n) = F(n - \sqrt{n}) + \sqrt{n}$;

By the anti-derivative method. $\Theta(n)$.

(i) $F(n) = F(n - \sqrt{n}) + 1$

By the anti-derivative method. $\Theta(\sqrt{n})$.

(j) $F(n) = F(3n/5) + F(4n/5) + n^2$

By Akra-Brazzi. $\Theta(n^2 \log n)$

(k) Duplicate eliminated

(l) $F(n) = F(12n/13) + F(5n/13) + n$

By Akra-Brazzi. $\Theta(n^2)$

(m) $F(n) = 2F(n-1) + 1$ **Think.**

This is the exponential one. $\Theta(2^n)$.

5. The following code computes the product of its two parameters. Write the loop invariant of the loop.

```
float product(float x, int n)
  // input condition: n >= 0
 {
  float y = x;
  int m = n;
  float z = 0.0;
  while(m > 0)
   {
    if(m%2) z = z+y;
    y = y+y;
    m = m/2;
   }
  return z;
 }
```

$n * x = m * y + z$

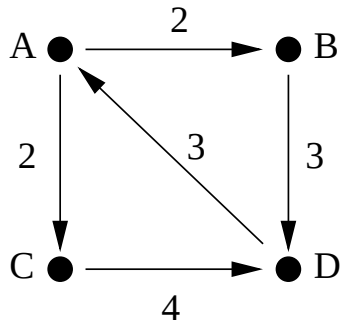
6. Write pseudocode for the Floyd-Warshall algorithm. Assume that the vertices are numbered from 1 to n, that W(i,j) is the weight of the arc from i to j, and that W(i,j) = ∞ if there is no arc from i to j. Be sure to compute back pointers.

    For all i and all j
    V[i,j] = W(i,j) and back[i,j] = i
For all i
    V[i,i] = 0
For all j
    For all i
        For all k
            temp = V[i,j]+V[j,k]
            if(temp ¡ V[i,k]
                V[i,k] = temp back[i,k] = back[j,k]

7. Use heapsort to sort the array QWERTYUIOP. Use the following matrix for your computation. Add extra rows as necessary.

Q	W	E	R	T	Y	U	I	O	P
Q	W	Y	R	T	E	U	I	O	P
Y	W	Q	R	T	E	U	I	O	P
Y	W	U	R	T	E	Q	I	O	P
P	W	U	R	T	E	Q	I	O	**Y**
W	P	U	R	T	E	Q	I	O	**Y**
W	T	U	R	P	E	Q	I	O	**Y**
O	T	U	R	P	E	Q	I	**W**	**Y**
U	T	Q	R	P	E	O	I	**W**	**Y**
I	T	Q	R	P	E	O	**U**	**W**	**Y**
T	I	Q	R	P	E	O	**U**	**W**	**Y**
T	R	Q	I	P	E	O	**U**	**W**	**Y**
O	R	Q	I	P	E	**T**	**U**	**W**	**Y**
R	O	Q	I	P	E	**T**	**U**	**W**	**Y**
R	P	Q	I	O	E	**T**	**U**	**W**	**Y**
E	P	Q	I	O	**R**	**T**	**U**	**W**	**Y**
Q	P	E	I	O	**R**	**T**	**U**	**W**	**Y**
O	P	E	I	**Q**	**R**	**T**	**U**	**W**	**Y**
P	O	E	I	**Q**	**R**	**T**	**U**	**W**	**Y**
I	O	E	**P**	**Q**	**R**	**T**	**U**	**W**	**Y**
O	I	E	**P**	**Q**	**R**	**T**	**U**	**W**	**Y**
E	I	**O**	**P**	**Q**	**R**	**T**	**U**	**W**	**Y**
I	E	**O**	**P**	**Q**	**R**	**T**	**U**	**W**	**Y**
E	**I**	**O**	**P**	**Q**	**R**	**T**	**U**	**W**	**Y**
E	**I**	**O**	**P**	**Q**	**R**	**T**	**U**	**W**	**Y**

8. (a) Finish the matrix W representing the weighted directed graph shown below. I have written the first two rows. (b) Use tropical matrix multiplication to compute W^3, which represents the graph of minimum path distances. Do not calculate backpointers.

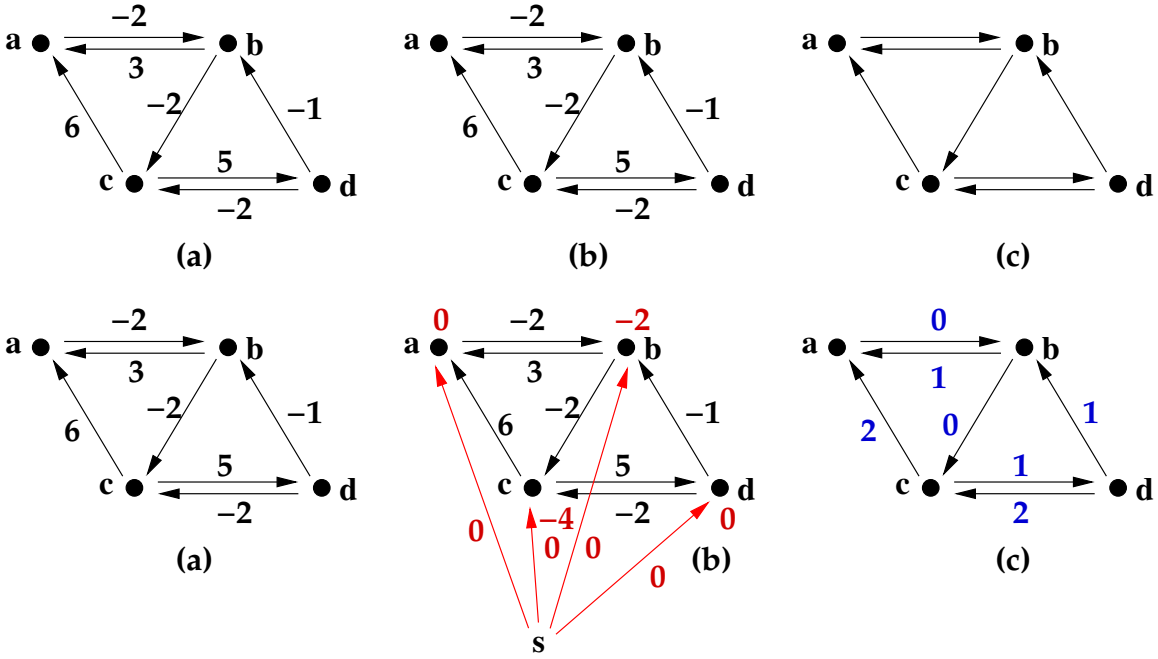


$W =$

	A	B	C	D
A	0	2	2	∞
B	∞	0	∞	3
C	∞	∞	∞	4
D	3	∞	∞	∞

9. Compute the first phase of Johnson's algorithm using the weighted graph (a) below. Use (b) for your intermediate work, and show the adjusted arc weights in (c). Do not finish Johnson's algorithm.

I failed to give an example. Here is one, with a solution.



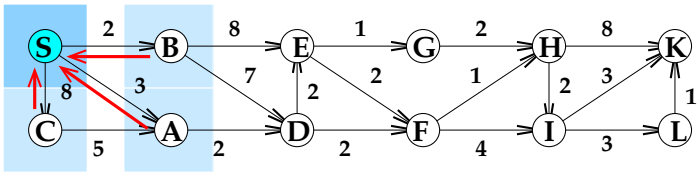
10. Solve the single source minpath problem using Dijkstra's algorithm for the weighted graph shown, where S is the source vertex. Fill in the matrix at each step, as was done for the example shown in the handout shown in class. Each edge shown is actually two arcs, one in each direction.

I have decided that the example I gave is too long. I have replaced it with a smaller example, which still requires 10 steps. At each step I list the vertices in the minheap in order. I have indicated the fully processed vertices, with a light blue background, and the fully processed vertices with darker blue. It is not necessary for you to have those backgrounds.

To help you get started, I have written the first two steps. I have added a page with two more copies of the graph and the matrix. To make grading easier, please do your work on copies of that page, as many

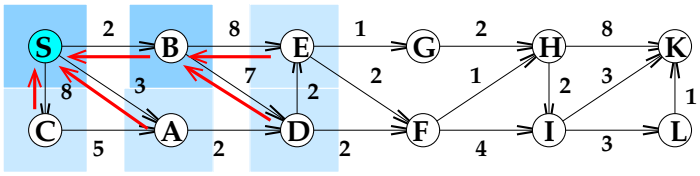
as you need.

Minheap = S



	S	A	B	C	D	E	F	G	H	I	K	L
V	0	3	2	8								
back		S	S	S								

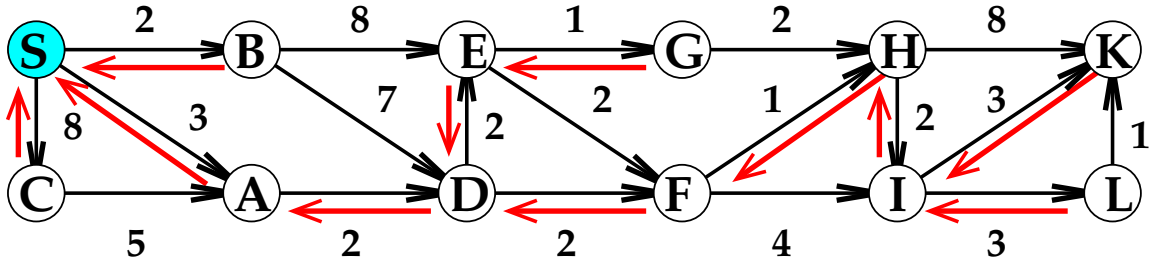
Minheap = BAC



	S	A	B	C	D	E	F	G	H	I	K	L
V	0	3	2	8	9	10						
back		S	S	S	B	B						

Minheap = ACDE

Here is the final figure.



	S	A	B	C	D	E	F	G	H	I	K	L
V	0	3	2	8	~~9~~ 5	~~10~~ 7	7	8	8	~~11~~ 10	~~16~~ 13	13
back		S	S	S	~~B~~ A	~~B~~ D	D	E	F	~~F~~ H	~~H~~ I	I