CS477 Final Examination December 11, 2024

Name:_____

No books, notes, scratch paper, or calculators. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided. If you want anything on extra pages to be graded, staple those pages to your test and write, "Please grade this page."

The entire examination is 500 points.

In any problem involving a graph or a directed graph, we assume that there are n vertices and m edges, or arcs, unless otherwise specified.

1. In each blank, write Θ if correct, otherwise write O or Ω, whichever is correct.

   (i) [5 points] $\log(n!) =$ _____ $(n \log n)$

   (ii) [5 points] $\log_2 n =$ _____ $(\log_4 n)$

2. [10 points] Evaluate $2^{\log_4 25}$.

   Hint: it's an integer.

3. True or False. Write "O" if the answer is not known to science at this time.

   (i) [5 points] _____ A good programmar would never use an unordered list. as a search structure.

   (ii) [5 points] _____ There is a mathematical statement which is true, yet cannot be proven.

   (iii) [5 points] _____ The subproblems of a dynamic program form a directed acyclic graph.

   (iv) [5 points] _____ Open hashing uses open addressing.

   (v) [5 points] _____ Treesort is an efficient implementation of insertion sort.

4. [5 points] Fill in the blanks.

   (i) [5 points] _____ is an efficient implementation of selection sort.

   (ii) [5 points] _____ algorithm does not allow the weight of any arc to be negative.

   (iii) [5 points] The asymptotic time complexity of Johnson's algorithm is _____. (Your answer should use O notation.)

   (iv) [5 points] The time complexity of every comparison-based sorting algorithm is _____ .

(v) [5 points] The items stored in a priority queue (that includes stacks, queues, and heaps) represent _____ _____.

(vi) [5 points] The asymptotic complexity of Dijkstra's algorithm algorithm is _____.

(vii) [5 points] A _____ hash function has no collisions.

(viii) [5 points] _____ and _____ are divide-and-conquer sorting algorithms that we've studied this semester.

(ix) [5 points] A spanning tree of a connected graph with 10 vertices and 14 edges has _____ edges. (Exact answer, please. No partial credit.)

(x) [5 points] The infix expression $-(a*b-c)$ is equivalent to the postfix expression _____.

(xi) [5 points] In _____ hashing, each item has more than one hash value, but only uses one of them.

5. Give the asymptotic complexity, in terms of `n`, of each of the following code fragments.

(i) [10 points]

```
for(int i = 2; i < n; i = i*i)
  cout << "Hello world" << endl;
```

(ii) [10 points]

```
for(int i = 1; i < n; i++)
  for(int j = 1; j < i; j = 2*j)
    cout << "Hello world" << endl;
```

(iii) [10 points]

```
for(int i = 1; i < n; i++)
  for(int j = i; j < n; j = 2*j)
    cout << "hello world" << endl;
```

Solve the recurrences. Give the asymptotic value of $F(n)$ in terms of n, using Θ notation.

(i) [10 points] $F(n) = F\left(\frac{n}{2}\right) + n$

(ii) [10 points] $F(n) = 2F\left(\frac{n}{2}\right) + n$

(iii) [10 points] $F(n) = 4F\left(\frac{n}{2}\right) + n$

(iv) [10 points] $F(n) = F\left(\frac{n}{2}\right) + 2F\left(\frac{n}{4}\right) + n$

(v) [10 points] $F(n) = 2F(n/2) + n^2$

(vi) [10 points] $F(n) = 3F(n/9) + 1$

(vii) [10 points] $F(n) = 4F(n/2) + n^2$

(viii) [10 points] $F(n) = F(\sqrt{n}) + 1$

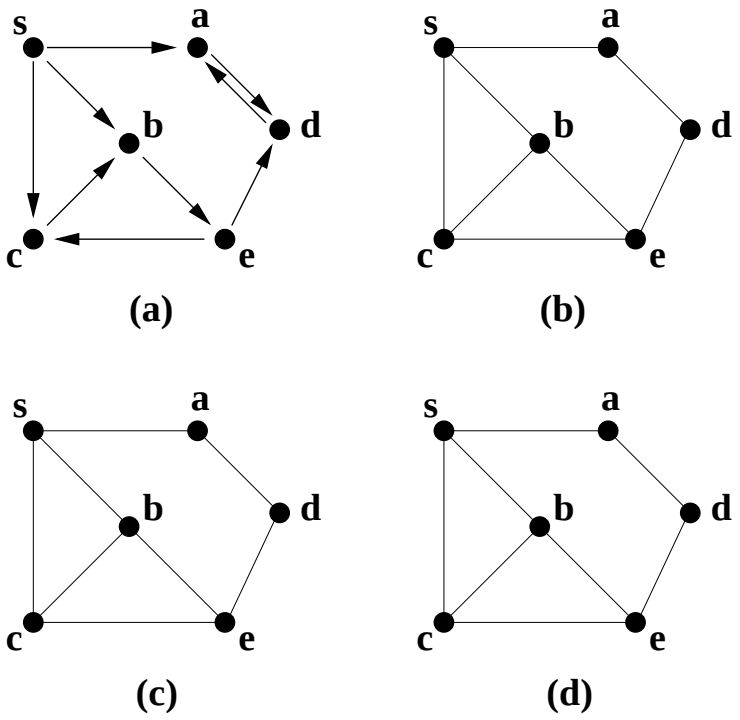
(ix) [10 points] $F(n) = F\left(\frac{3n}{5}\right) + 4F\left(\frac{2n}{5}\right) + n^2$

(x) [10 points] $F(n) = 3F\left(\frac{n}{3}\right) + 3F\left(\frac{2n}{3}\right) + n^2$

(xi) [10 points] $F(n) = 2F\left(\frac{n}{4}\right) + \sqrt{n}$

(xii) [10 points] $F(n) = F(\log n) + 1$

6. [20 points] Use the DFS method to find the strong components of the digraph shown in (a) below. Use (b), (c), and (d) to show your work.



7. [20 points] You are given an acyclic directed graph $G = (V, E)$ Describe a dynamic programming algorithm which calculates the maximum number of edges of any directed path through G. Let $\text{Innbr}(x)$ be the set of in-neighbors of a vertex x, and $\text{Outnbr}(x)$ its set of out-neighbors.

8. [10 points] List properties of a good hash function.

9. Consider the following C++ code.

```
int martha(int n)
 {
  if(n <= 1) return 1;
  else return martha(n/2)+martha((n+1)/2)+n;
 }
```

   (i) [10 points] What is the asymptotic complexity of `martha(n)`?

   (ii) [10 points] What is the time complexity of the recursive code given above?

   (iii) [10 points] What is the space complexity of a computation of `martha(n)` using memoization?

10. Fill in the blanks.

   (i) [5 points] The asymptotic expected height of a treap with n nodes is _____.

   (ii) [5 points] If G is a weighted digraph, it is impossible to solve any shortest path problem on G if G has a _____ _____

(iii) [5 points] The height of a binary tree with 45 nodes is at least _____. (You must give the exact answer. No partial credit.)

(iv) [5 points] The following is pseudo-code for what algorithm? _____

```
int x[n];
input values of x;
 for(int i = n-1; i > 0; i--)
  for(int j = 0; j < i; j++)
   if(x[i] < x[j]) swap(x[i],x[j]);
```

(v) [5 points] In closed hashing, if the position at $h(x)$ is already occupied for some data item x, a _____ sequence is used to find an unoccupied position in the hash table.

11. [10 points] What is the loop invariant of the loop in the following function?

```
float product(float a, int n)
 {
  // assert(n >= 0);
  float c = 0.0;
  float b = a;
  int p = n;
  while(m > 0)
   {
    if(p%2) c = c+b;
    p = p/2;
    b = b+b;
   }
  return c;
 }
```

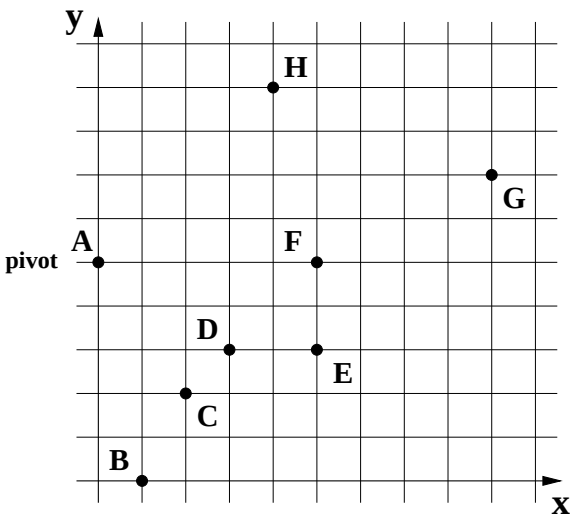
12. [20 points] Write pseudo-code for the Floyd/Warshall algorithm. Let the vertices be $\{1, 2, \ldots n\}$. Let $W(i, j)$ be the given weight of the arc (i, j), if any, where $W(i, j) = \infty$ if there is no arc. Compute $V(i, j)$, the minimum weight of any path from i to j, and $B(i, j)$, the backpointer for that minimum path.

13. [20 points] A compiler stores an array `A[9][5][12]` into main memory in row major order, with base address 1024. Each entry of A requires one unit of memory. What is the address of `A[5][3][7]` in main memory?

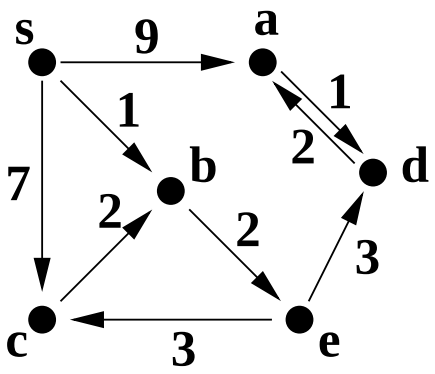
14. [20 points] Consider an array implementation of a stack of integers, as given below. Fill in the code which implements the needed operators of a stack.

```
const int N = // whatever
struct stack
 {
  int item[N];
  int size; // number of items in the stack
  // bottom of the stack is at item[0];
 };
void initialize(s&stack)
 {

 }
void push(s&stack,int i)
 {



 }
 bool empty{s&stack)
  {

  }
 int pop(s&stack)
  {



  }
```

15. [20 points] Using an algorithm we discussed in class, find the convex hull of the set of points indicated in the figure below. Choose A to be the pivot. Show your steps.



16. [20 points] Use Dijkstra's algorithm to solve the single source shortest path problem for the following weighted directed graph, where **s** is the source. Show the steps.



17. [10 points] Find an optimal prefix code for the alphabet $\{a, b, c, d, e, f\}$ where the frequencies are given in the following array.

a	7
b	3
c	5
d	8
e	12
f	1

18. [20 points] Write pseudcode for the simple coin-row problem we discussed in class. You are given a row of n coins of various values. The problem is to select a set of coins of maximum total value, subject to the condition that no two adjacent coins are selected. Your code should identify the coins which are selected.