CS477 Final Examination May 6, 2024

500 total points

No books, notes, or electronic devices, or scratch paper. Scratch paper will be provided. Write, "Grade this page," and your name, on any scratch paper you want graded and staple it to the test. Also write, "See scratch paper," on the test at the appropriate place.

For any complexity question involving a graph or digraph, let n be the number of vertices and m the number of edges or arcs.

In any time complexity question, assume that each arithmetic operation, such as addition or multiplication, takes constant time.

1. In each blank, write Θ if correct, otherwise write O or Ω, whichever is correct.

   (i) [5 points] $2^n =$ ________ (n^2)

   (ii) [5 points] $\log n^2 =$ ________ $(\log n^3)$

   (iii) [5 points] $\log^2 n =$ ________ $(\log n)$

   (iv) [5 points] $3^{n/2} =$ ________ (2^n)

2. True or False. Write "O" if the answer is not known to science at this time.

   (i) [5 points] ________ A dynamic program with n subproblems, each of which an be worked in constant time, can always be worked in polylogarithmic time by dividing up the task among polynomially many processors working in parallel.

   (ii) [5 points] ________ No good programmar would ever implement a search structure as an unordered list.

   (iii) [5 points] ________ Computers are so fast nowadays that there is no longer any point to analyzing the time complexity of a program.

   (iv) [5 points] ________ Closed hashing uses open addressing.

3. Fill in the blanks.

   (i) [5 points] The worst case asymptotic complexity of heapsort is ________________________________.

   (ii) [5 points] The subproblems of a dynamic program must be worked in ________________________________ order.

   (iii) [5 points] The height of a binary tree with 28 nodes is at least ________________. (You must give the best possible answer, exactly. No partial credit.)

   (iv) [5 points] ________________________________ algorithm does not allow the weight of any arc to be negative.

   (v) [5 points] The time complexity of every comparison-based sorting algorithm is ________________________ . (Your answer should use Ω notation.)

(vi) [5 points] The items stored in a priority queue (that includes stacks, queues, and heaps) represent ________________ ________________.

(vii) [5 points] The asymptotic complexity of the Floyd/Warshall algorithm is ______________________.

(viii) [5 points] The asymptotic complexity of Dijkstra's algorithm algorithm is ______________________.

(ix) [5 points] The asymptotic worst case time complexity of Johnson's algorithm is ______________________ . (Your answer should use O notation.)

(x) [5 points] A ______________________ hash function fills the hash table exactly with no collisions.

(xi) [5 points] ______________________ algorithm finds a binary code so that the code for one symbol is never a prefix of the code for another symbol.

(xii) [5 points] The asymptotic expected time to find the median item in an unordered array of size n, using a randomized selection algorithm, is ___________.

(xiii) [5 points] If a planar graph has 10 edges, it must have at least ___________ vertices.

(xiv) [10 points] The prefix expression $*+\sim ab\sim -cd$ is equivalent to the infix expression

____________________________ and the postfix expression ____________________________.

(xv) [5 points] In ________________ hashing, each item has more than one hash value, but only uses one of them.

4. For each of the 4 blanks below, write one of these answers. **selection sort**, **insertion sort**, **bubble sort**, **depth first search**, **breadth first search**.

(i) [5 points] Heapsort is a fast form of ______________________.

(ii) [5 points] Treesort is a fast form of ______________________.

(iii) [5 points] Dijkstra's Algorithm is a modification of ______________________.

5. Give the asymptotic complexity, in terms of `n`, of each of the following code fragments.

(i) [5 points]

```
for(i = 0; i < n; i = i+1);
cout << "Hello world!" << endl;
```

(ii) [5 points]

```
for(int i = 1; i < n; i = i+i)
  cout << "Hello world" << endl;
```

(iii) [5 points]

```
for(int i = 1; i < n; i++)
  for(int j = 1; j < i; j = 2*j)
    cout << "Hello world" << endl;
```

(iv) [5 points]

```
for(int i = 1; i < n; i++)
  for(int j = i; j < n; j = 2*j)
    cout << "hello world" << endl;
```

(v) [5 points]

```
for(int i = 1; i*i < n; i++)
  cout << "hello world" << endl;
```

(vi) [5 points]

```
for(int i = 0; i < n; i++)
  for(int j = n; j > i; j = j/2)
```

(vii) [5 points]

```
for(int i = 0; i < n; i++)
  for(int j = i; j > 0; j = j/2)
```

(viii) [5 points]

```
for(int i = 2; i < n; i=i*i)
  cout << "Hello world!" << endl;
```

(ix) [5 points]

```
for(int i = 1; i < n; i++)
  for(int j = 2; j < i; j=j*j)
    cout << "Hello world" << endl;
```

6. Solve the recurrences. Give the asymptotic value of $F(n)$ in terms of n, using Θ notation.

(i) [5 points] $F(n) = F\left(\frac{n}{2}\right) + n$

(ii) [5 points] $F(n) = 2F\left(\frac{n}{2}\right) + n$

(iii) [5 points] $F(n) = 4F\left(\frac{n}{2}\right) + n$

(iv) [5 points] $F(n) = F\left(\frac{n}{2}\right) + 2F\left(\frac{n}{4}\right) + n$

(v) [5 points] $F(n) = 2F(n/2) + n^2$

(vi) [5 points] $F(n) = 3F(n/9) + 1$

(vii) [5 points] $F(n) = 4F(n/2) + n^2$

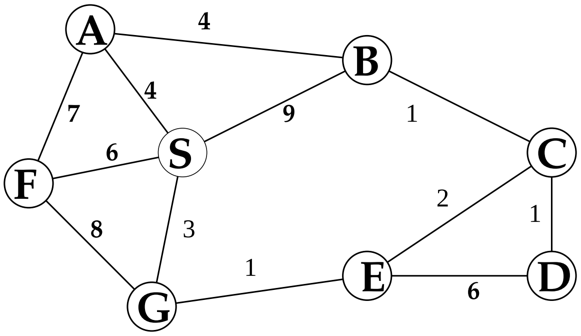
(viii) [5 points] $F(n) = F(\sqrt{n}) + 1$

(ix) [5 points] $F(n) = F(3n/5) + 4F(2n/5) + n^2$

(x) [5 points] $F(n) = 2F(n/4) + \sqrt{n}$

7. [20 points] Explain how to compute the n^{th} Fibonacci number $F(n)$ in $O(\log n)$ time using the recurrence
$F(1) = F(2) = 1$
$F(n) = F\left(\frac{n-1}{2}\right) * F\left(\frac{n}{2}\right) + F\left(\frac{n+1}{2}\right) * F\left(\frac{n+2}{2}\right)$ for $n > 2$
where the quotient of integers is truncated to an integer, as in C++.

8. [20 points] Use Dijkstra's algorithm to solve the single source shortest path problem for the following weighted directed graph, where **S** is the source. Show the steps.



9. [10 points] Find an optimal prefix code for the alphabet $\{a, b, c, d, e, f\}$ where the frequencies are given in the following array.

a	3
b	6
c	4
d	5
e	8
f	9

10. [10 points] What is the loop invariant of the loop in the following function?

```
float product(float x, int n)
 {
  // assert(n >= 0);
  float z = 0.0;
  float y = x;
  int m = n;
  while(m > 0)
   {
    if(m%2) z = z+y;
    m = m/2;
    y = y+y;
   }
  return z;
 }
```

11. [10 points] Compute the Levenshtein distance between abcdba and acdbce. Show the matrix.

12. [20 points] You need to store Pascal's triangle in row-major order into a 1-dimensional array P whose indices start at 0. The triangle is infinite, but you will only store $\binom{n}{k}$ for $n < N$. Write a function I such that $P[I(n,k)] = \binom{n}{k}$ for $0 \leq k \leq n < N$. For example, $I(3,2) = 8$.

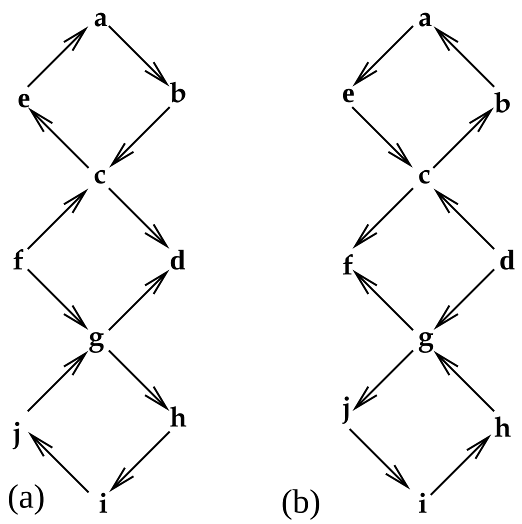
```
        1
      1   1
    1   2   1
  1   3   3   1
1   4   6   4   1
```

```
int I(int n, int k)
 {
  // the position of n choose k in the linear array
  assert(k >= 0 and n >= k and n < N);
  int indx =



  return indx;
 }
```

13. [20 points] Use the DFS method to find the strong components of the digraph shown below as (a). (b) is the inverse digraph. Show your steps.



14. [10 points] Sketch a circular linked list with dummy node which implements a queue. The queue has four items. From front to rear, these are A, B, C, D, and show the insertion of E into the queue. Show the steps. Don't erase deleted links; instead, simply cross them out.

15. [10 points] Write pseudocode for the Bellman-Ford algorithm. Be sure to include the shortcut.

16. [10 points] List properties of a good hash function.

17. [10 points] Walk through mergesort with the array given below.

```
XORWYFHBVESUP
```

18. [10 points] You are given a row of coins of values, $x_1, x_2, \ldots x_n$. Give a dynamic program which computes the maximum total value of a set of coins no two of which are adjacent in the original row. Your program does not have to print the optimal set.

19. Fill in the blanks.

(i) [5 points] The worst case asymptotic time complexity of treesort is ________________ if an AVL tree is used.

(ii) [5 points] The asymptotic expected height of a treap with n nodes is ________________.

(iii) [5 points] The worst case asymptotic time complexity of quicksort is ________________

(iv) [5 points] If G is a weighted digraph, it is impossible to solve any shortest path problem on G if G has a ______________________________ ______________________________

(v) [5 points] The following is pseudo-code for what algorithm? ______________________________

```
int x[n];
input values of x;
 for(int i = n-1; i > 0; i--)
  for(int j = 0; j < i; j++)
   if(x[i] < x[j]) swap(x[i],x[j]);
```

(vi) [5 points] In closed hashing, if the position at $h(x)$ is already occupied for some new data item x, a ______________________ sequence is used to find an unoccupied position in the hash table.

(vii) [5 points] A planar graph with $n \geq 3$ vertices can have no more than _______________ edges. (Exact formula, please.)

20. [10 points] Write pseudo-code for the Floyd/Warshall algorithm. Let the vertices be $\{1, 2, \ldots n\}$. Let $W(i, j)$ be the given weight of the arc (i, j), if any, where $W(i, j) = \infty$ if there is no arc. Compute $V(i, j)$, the minimum weight of any path from i to j, and $B(i, j)$, the backpointer for that minimum path.

21. [5 points] Walk through polyphase mergesort with the array given below.

```
ACPREHJMQSOZTUNXVB
```

22. [20 points] A compiler stores an array `A[7][11][12]` into main memory in column major order, with base address B, and each entry of A requires one place in main memory. Write a formula for the main memory address of `A[i][j][k]` for integers i, j, and k within range.

23. [20 points] Consider an array implementation of a stack of integers, as given below. Fill in the code which implements the needed operators of a stack.

```
const int N = // whatever
struct stack
 {
  int item[N];
  int size; // number of items in the stack
  // bottom of the stack is at item[0];
 };
void initialize(s&stack)
 {


 }
void push(s&stack,int i)
 {






 }
 bool empty{s&stack)
  {


  }
```

```
int pop(s&stack)
 {





 }
```

24. [5 points] In class, we implemented a minheap as an almost complete binary tree implemented as an array. Suppose the minheap is initialized as shown in the first line of the array shown below. Show the evolution of the structure when deletemin is executed.

A	C	K	E	H	Q	N	M	G	S

25. [5 points] Starting from the configuration given, show the evolution of the structure when B is inserted.

C	E	K	G	H	Q	N	M	S	

26. [10 points] Using one of the algorithm we discussed in class, find the convex hull of the set of points indicated in the figure below. Show your steps.