CS477 Final Examination May 14, 2025

The entire examination is 510 points.

No books, notes, electronic devices, or scratch paper. Scratch paper will be provided. Write, "Grade this page," and your name, on any scratch paper you want graded, and staple it to the test. Also write, "See scratch paper," on the test at the appropriate place.

For any complexity question involving a graph or digraph, let n be the number of vertices and m the number of edges or arcs.

In any time complexity question, assume that each arithmetic operation, such as addition or multiplication, takes constant time.

1. True or False. Write "O" if the answer is not known to science at this time.

   (a) [5 points] _____ A dynamic program with n subproblems, each of which can be worked in constant time, can always be worked in polylogarithmic time by dividing up the task among polynomially many processors working in parallel.

   (b) [5 points] _____ Open hashing uses open addressing.

2. In each blank, write Θ if correct, otherwise write O or Ω, whichever is correct.

   (i) [5 points] $\log n^2 =$ _____ $(\log n^3)$

   (ii) [5 points] $\log^2 n =$ _____ $(\log n)$

   (iii) [5 points] $\log(n!) =$ _____ $(n \log n)$

   (iv) [5 points] $\log n^2 =$ _____ $(\log n^3)$

   (v) [5 points] $\log_3 5 =$ (_____) $(\log_4 5)$

3. Fill in the blanks.

   (a) [5 points] The asymptotic expected height of a treap with n nodes is _____.

   (b) [5 points] The worst case asymptotic time complexity of quicksort is _____

   (c) [5 points] If G is a weighted digraph, it is impossible to solve any shortest path problem on G if G has a _____ _____

   (d) [5 points] In closed hashing, if the position at $h(x)$ is already occupied for some new data item x, a _____ sequence is used to find an unoccupied position in the hash table.

   (e) [5 points] A planar graph with $n \geq 3$ vertices can have no more than _____ edges. (Exact formula, please.)

   (f) [5 points] A binary tree with height h cannot have more than _____ leaves. (Exact formula, please.)

   (g) [5 points] _____ algorithm does not allow the weight of any arc to be negative.

(h) [5 points] The time complexity of every comparison-based sorting algorithm is _____ . (Your answer should use Ω notation.)

(i) [5 points] The asymptotic complexity of the Floyd/Warshall algorithm is _____.

(j) [5 points] The asymptotic complexity of Dijkstra's algorithm is _____. (Your answer should use O notation.)

(k) [5 points] The asymptotic time complexity of Johnson's algorithm is _____. (Your answer should use O notation.)

(l) [5 points] A _____ hash function fills the hash table exactly with no collisions.

(m) [5 points] The asymptotic expected time to find the median item in an unordered array of size n, using a randomized selection algorithm, is _____.

(n) [10 points] The prefix expression $*+ \sim ab \sim -cd$ **CHANGE** is equivalent to the infix expression _____ and the postfix expression _____.

(o) [5 points] In _____ hashing, each item has more than one hash value, but only uses one of them.

4. For each of the 4 blanks below, write one of these answers. **selection sort**, **insertion sort**, **bubble sort**, **depth first search**, **breadth first search**.

(a) [5 points] Heapsort is a form of _____.

(b) [5 points] Treesort is a form of _____.

(c) [5 points] Dijkstra's algorithm is a modification of _____.

(d) [5 points] The A* algorithm is a modification of _____.

5. Solve the recurrences. Give the asymptotic value of $F(n)$ in terms of n, using Θ notation.

(a) [5 points] $F(n) = F\left(\frac{n}{2}\right) + n$

(b) [5 points] $F(n) = 2F\left(\frac{n}{2}\right) + n^2$

(c) [5 points] $F(n) = 4F\left(\frac{n}{2}\right) + n$

(d) [5 points] $F(n) = F\left(\frac{n}{2}\right) + 2F\left(\frac{n}{4}\right) + n$

(e) [5 points] $F(n) = F(\sqrt{n}) + 1$

(f) [5 points] $F(n) = F(3n/5) + 4F(2n/5) + n^2$

6. Give the asymptotic complexity, in terms of `n`, of each of the following code fragments.

(i) [5 points]

```
for(int i = 0; i < n; i++)
  for(int j = i; j > 0; j = j/2)
```

(ii) [5 points]

```
for(int i = 2; i < n; i = i+i)
  cout << "Hello world" << endl;
```

(iii) [5 points]

```
for(int i = 1; i*i < n; i++)
  cout << "hello world" << endl;
```

(iv) [5 points]

```
for(int i = 0; i < n; i++)
  for(int j = n; j > i; j = j/2)
```

(v) Duplicate

```
for(int i = 0; i < n; i++)
  for(int j = i; j > 0; j = j/2)
```

(vi) [5 points]

```
for(int i = 2; i < n; i=i*i)
  cout << "Hello world!" << endl;
```

(vii) [5 points]

```
for(int i = 1; i < n; i++)
  for(int j = 2; j < i; j=j*j)
    cout << "Hello world" << endl;
```

7. [20 points] Explain how to compute the n^{th} Fibonacci number $F(n)$ in $O(\log n)$ time using the recurrence
$F(1) = F(2) = 1$
$F(n) = F\left(\frac{n-1}{2}\right) * F\left(\frac{n}{2}\right) + F\left(\frac{n+1}{2}\right) * F\left(\frac{n+2}{2}\right)$ for $n > 2$
where, for an integer n, the expression $n/2$ is always rounded down to an integer, as in C++. Be sure to state the asymptotic space complexity of your algorithm.

8. [10 points] Find an optimal prefix code for the alphabet $\{a, b, c, d, e, f\}$ where the frequencies are given in the following array.

a	3
b	6
c	4
d	5
e	8
f	9

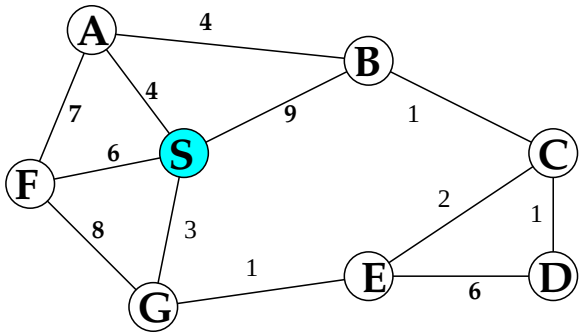
9. [10 points] Compute the Levenshtein distance between bdef and abce. Show the matrix.

10. [10 points] What is the loop invariant of the loop in the following function?

```
float product(float x, int n)
```

```
{
 // assert(n >= 0);
 float z = 0.0;
 float y = x;
 int m = n;
 while(m > 0)
  {
   if(m%2) z = z+y;
   m = m/2;
   y = y+y;
  }
 return z;
}
```

11. [20 points] Use Dijkstra's algorithm to solve the single source shortest path problem for the following weighted directed graph, where **S** is the source. Show the steps.



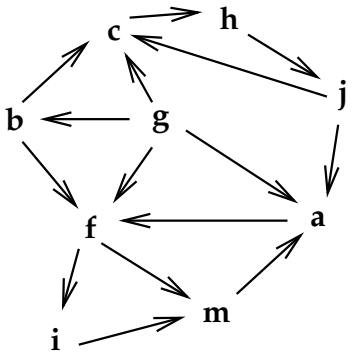
12. [20 points] Store Pascal's triangle in row-major order as a 1-dimensional array P whose indices start at 0. The triangle is infinite, but you will only store $\binom{n}{k}$ for $n < N$. Write a function I such that $P[I(n,k)] = \binom{n}{k}$ for $0 \le k \le n < N$. For example, $I(3,2) = 8$.

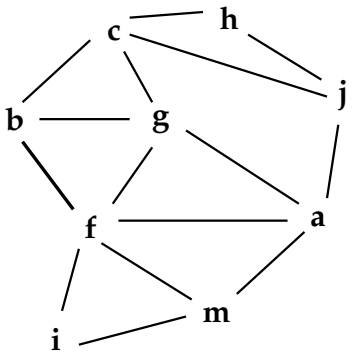
```
                1
              1   1
            1   2   1
          1   3   3   1
        1   4   6   4   1
```

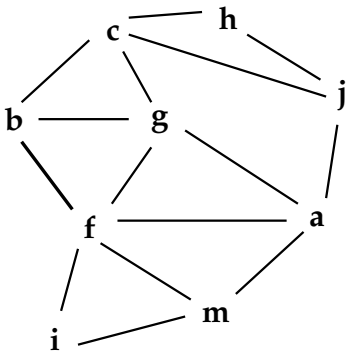
```
int I(int n, int k)
 {
  // the position of n choose k in the linear array
  assert(k >= 0 and n >= k and n < N);
  int indx =
```

```
 return indx;
}
```

13. [20 points] Use the DFS method to find the strong components of the digraph shown on the left. Use the other figures for your work, as needed.







14. [10 points] Write pseudocode for the Bellman-Ford algorithm. Be sure to include the shortcut.

15. [10 points] List properties of a good hash function.

16. [20 points] The following code computes an integer approximation to a well-known function of n. Circle that function.

n $\log_2 n$ $\sqrt{n}$ n^2 $\frac{n}{2}$ $n^{3/2}$ $n \log n$

```
int mystery(int n)
// input condition: n > 0
  {
   int lo = 0;
   int hi = n;
   while(lo+1 < hi)
    {
     int mid = (lo+hi)/2;
     if (n*n < mid) hi = mid;
     else lo = mid;
    }
   return lo;
  }
```

Hint: the code uses binary search.

17. [20 points] Consider an array implementation of a stack of integers, as given below. Fill in the code which implements the needed operators of a stack.

```
const int N = // whatever
struct stack
 {
  int item[N];
  int size; // number of items in the stack
  // bottom of the stack is at item[0];
 };
void initialize(s&stack)
 {


 }
void push(s&stack,int i)
 {







 }
 bool empty{s&stack)
  {


  }
 int pop(s&stack)
  {




  }
```

18. [10 points] Walk through mergesort with the array given below.

    XORWYFHBVESUP

19. [10 points] Write pseudo-code for the Floyd/Warshall algorithm. Let the vertices be $\{1, 2, \ldots n\}$. Let $W(i, j)$ be the given weight of the arc (i, j), if any, where $W(i, j) = \infty$ if there is no arc. Compute $V(i, j)$, the minimum weight of any path from i to j, and $B(i, j)$, the backpointer for that minimum path.

20. [20 points] You are given a row of coins of values, $x_1, x_2, \ldots x_n$. Give a dynamic program which computes the maximum total value of a set of coins no two of which are adjacent in the original row. Your program does not have to print the optimal set.

21. [5 points] Walk through polyphase mergesort with the array given below.

    ACPREHJMQSOZTUNXVB

22. [20 points] A compiler stores an array `A[7][11][12]` into main memory in column major order, with base address B, and each entry of A requires one place in main memory. Write a formula for the main memory address of `A[i][j][k]` for integers i, j, and k within range.

23. [10 points] Find the maximum length strictly monotone subsequence of 4, 9, 2, 7, 3, 6, 5. Show your work.

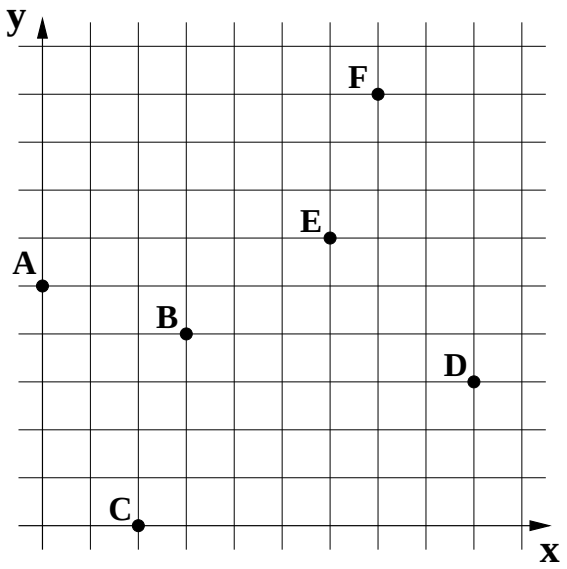
24. [5 points] In class, we implemented a minheap as an almost complete binary tree implemented as an array. Suppose the minheap is initialized as shown in the first line of the array shown below. Show the evolution of the structure when deletemin is executed.

A	C	K	E	H	Q	N	M	G	S

25. [5 points] Starting from the configuration given, show the evolution of the structure when B is inserted.

C	E	K	G	H	Q	N	M	S	

26. [10 points] Using the Graham scan algorithm, find the convex hull of the set of points indicated in the figure below. Show your steps.



27. [20 points] What are the asymptotic time and space complexities, in terms of n, of the code below. (Hint: they are not the same.) Assume $n \geq 0$.

```
int voltaire(int n)
 {
  if(n == 0) return 1;
  else return voltaire(n/2) + voltaire(n/2) + 1;
 }
```

28. [10 points] Sketch a circular linked list with dummy node which implements a queue. The queue has four items. From front to rear, these are R, Q, T, L, and show the insertion of A into the queue. Show the steps. Don't erase deleted links; instead, simply cross them out.

29. [20 points] Walk through the A* algorithm to find the shortest path from S to T in the weighted graph shown below. The heuristices are indicated by red numerals.