

Name: _____

Problem 3–3 Ordering by asymptotic growth rates

a. Rank the following functions by order of growth; that is, find an arrangement of the functions $g_1, g_2, \dots, g_{30}$ such that $g_1 = \Omega(g_2), g_2 = \Omega(g_3), \dots, g_{29} = \Omega(g_{30})$. Partition your list into equivalence classes such that any $f(n)$ and $g(n)$ are in the same class if and only if $f(n) = \Theta(g(n))$.

$\lg(\lg^* n)$	$2^{\lg^* n}$	$(\sqrt{2})^{\lg n}$	n^2	$n!$	$(\lg n)!$
$(\frac{3}{2})^n$	n^3	$\lg^2 n$	$\lg(n!)$	2^{2^n}	$n^{1/\lg n}$
$\ln \ln n$	$\lg^* n$	$n \cdot 2^n$	$n^{\lg \lg n}$	$\ln n$	1
$2^{\lg n}$	$(\lg n)^{\lg n}$	e^n	$4^{\lg n}$	$(n+1)!$	$\sqrt{\lg n}$
$\lg^*(\lg n)$	$2^{\sqrt{2 \lg n}}$	n	2^n	$n \lg n$	$2^{2^{n+1}}$

Exercise 4.1–1

Show that the solution of $T(n) = T(\lceil n/2 \rceil) + 1$ is $O(\lg n)$.

Exercise 4.6–1

Solve the recurrence $T(n) = 2T(\sqrt{n}) + 1$ by making a change of variables. Your solution should be asymptotically tight. Do not worry about whether values are integral.

Exercise 4.3–1

Use the master method to give tight asymptotic bounds for the following recurrences.

a. $T(n) = 4T(n/2) + n$.

b. $T(n) = 4T(n/2) + n^2$.

c. $T(n) = 16T(n/2) + n^3$.

Problem 4.1 Recurrence examples

Give asymptotic upper and lower bounds for $T(n)$ in each of the following recurrences. Assume that $T(n)$ is constant for $n \leq 2$. Make your bounds as tight as possible, and justify your answers.

a. $T(n) = 2T(n/2) + n^3$

b. $T(n) = 2T(9n/10) + n$

c. $T(n) = 2T(n/4) + n^2$

d. $T(n) = 7T(n/3) + n^2$

e. $T(n) = 7T(n/2) + n^2$

f. $T(n) = 2T(n/4) + \sqrt{n}$

g. $T(n) = T(n - 1) + n$

h. $T(n) = T(\sqrt{n}) + 1$