UNLV Computer Science 477/677 Spring 2011

Name: _____

Problem 3–3 Ordering by asymptotic growth rates

a. Rank the following functions by order of growth; that is, find an arrangement of the functions $g_1, g_2, \ldots g_{30}$ such that $g_1 = \Omega(g_2), g_2 = \Omega(g_3), \ldots g_{29} = \Omega(g_{30}$. Partition your list into equivalence classes such that any $f(n)$ and $g(n)$ are in the same class if and only if $f(n) = \Theta(g(n))$.

$\lg(\lg^* n)$	$2^{\lg^* n}$	$\left(\sqrt{2}\right)^{\lg n}$	n^2	$n!$	$(\lg n)!$
$\left(\frac{3}{2}\right)^n$	n^3	$\lg^2 n$	$\lg(n!)$	2^{2^n}	$n^{1/\lg n}$
$\ln\ln n$	$\lg^* n$	$n \cdot 2^n$	$n^{\lg\lg n}$	$\ln n$	1
$2^{\lg n}$	$(\lg n)^{\lg n}$	e^n	$4^{\lg n}$	$(n+1)!$	$\sqrt{\lg n}$
$\lg^*(\lg n)$	$2^{\sqrt{2\lg n}}$	n	2^n	$n \lg n$	$2^{2^{n+1}}$

Exercise 4.1–1

Show that the solution of $T(n) = T(\lceil n/2 \rceil) + 1$ is $O(\lg n)$.

Exercise 4.6–1

Solve the recurrence $T(n) = 2T(\sqrt{n}) + 1$ by making a change of variables. Your solution should be asymptotically tight. Do not worry about whether values are integral.

Exercise 4.3–1

Use the master method to give tight asymptotic bouds for the following recurrences.

a. $T(n) = 4T(n/2) + n$.

b. $T(n) = 4T(n/2) + n^2$.

c. $T(n) = 16T(n/2) + n^3$.

Problem 4.1 Recurrence examples

Give asymptotic uper and lower bounds for $T(n)$ in each of the following recurrences. Assume that $T(n)$ is constant for $n \leq 2$. Make your bounds as tight as possible, and justify your answers.

a. $T(n) = 2T(n/2) + n^3$

b. $T(n) = 2T(9n/10) + n$

c. $T(n) = 2T(n/4) + n^2$

d. $T(n) = 7T(n/3) + n^2$

e. $T(n) = 7T(n/2) + n^2$

f. $T(n) = 2T(n/4) + \sqrt{n}$

g. $T(n) = T(n-1) + n$

h. $T(n) = T(\sqrt{n}) + 1$