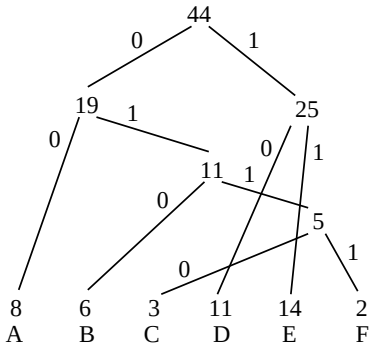
University of Nevada, Las Vegas Computer Science 477/677 Spring 2025

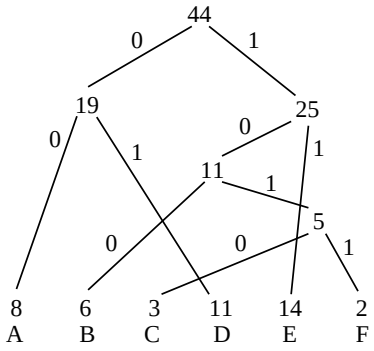
Answers to Assignment 4: Due Saturday March 1, 2025

1. Find an optimal Huffman code on the alphabet {A,B,C,D,E,F}. Frequencies are given in the table below.

   At one step, we combine 8 with a tree of weight 11, and there are two choices. Both are shown.

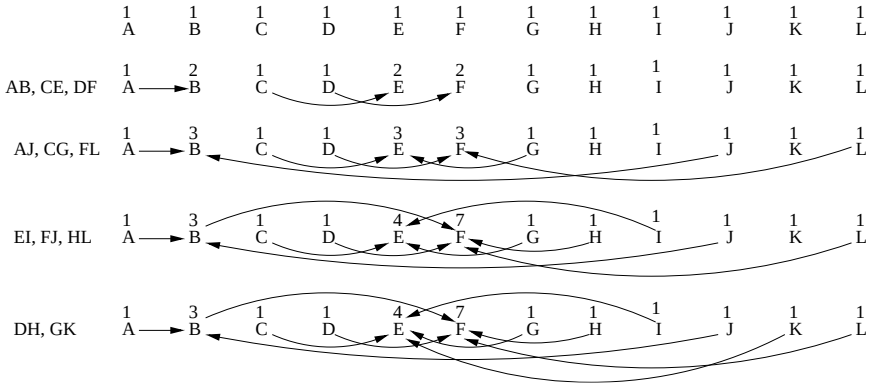
A	8	00	00
B	6	010	100
C	3	0110	1010
D	11	10	01
E	14	11	11
F	2	0111	1011



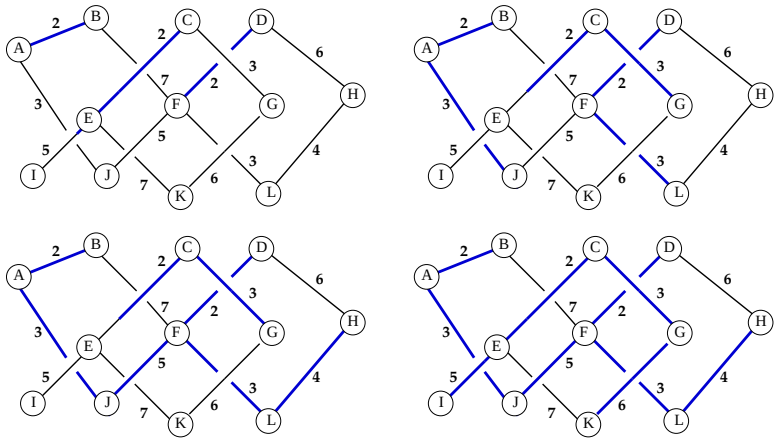


2. The weighted graph below has more than one component. Use Kruskal's algorithm to construct a subgraph which consists of a minimal spanning tree of each component.

   Evolution of the directed forest of the union-find algorithm.



   Evolution of the spanning forest of Kruskal's algorithm.



3. Walk through heapsort, using the method which has a *heapify* phase, for the array: **A Q R B X S M L N T** To make the problem easier to grade, please show the evolution of the array in the table below. If more rows are needed, draw them in. Indicate the step at which heapify has been completed.

1	2	3	4	5	6	7	8	9	10	
A	Q	R	B	X	S	M	L	N	T	
A	Q	R	N	X	S	M	L	B	T	
A	Q	S	N	X	R	M	L	B	T	
A	X	S	N	Q	R	M	L	B	T	
A	X	S	N	T	R	M	L	B	Q	
X	A	S	N	T	R	M	L	B	Q	
X	T	S	N	A	R	M	L	B	Q	
X	T	S	N	Q	R	M	L	B	A	heapify done
A	T	S	N	Q	R	M	L	B	**X**	
T	A	S	N	Q	R	M	L	B	**X**	
T	Q	S	N	A	R	M	L	B	**X**	heap order restored
B	Q	S	N	A	R	M	L	**T**	**X**	
S	Q	B	N	A	R	M	L	**T**	**X**	
S	Q	R	N	A	B	M	L	**T**	**X**	heap order restored
L	Q	R	N	A	B	M	**S**	**T**	**X**	
R	Q	L	N	A	B	M	**S**	**T**	**X**	
R	Q	M	N	A	B	L	**S**	**T**	**X**	heap order restored
L	Q	M	N	A	B	**R**	**S**	**T**	**X**	
Q	L	M	N	A	B	**R**	**S**	**T**	**X**	
Q	N	M	L	A	B	**R**	**S**	**T**	**X**	heap order restored
B	N	M	L	A	**Q**	**R**	**S**	**T**	**X**	
N	B	M	L	A	**Q**	**R**	**S**	**T**	**X**	
N	L	M	B	A	**Q**	**R**	**S**	**T**	**X**	heap order restored
A	L	M	B	**N**	**Q**	**R**	**S**	**T**	**X**	
M	L	A	B	**N**	**Q**	**R**	**S**	**T**	**X**	heap order restored
B	L	A	**M**	**N**	**Q**	**R**	**S**	**T**	**X**	
L	B	A	**M**	**N**	**Q**	**R**	**S**	**T**	**X**	heap order restored
A	B	**L**	**M**	**N**	**Q**	**R**	**S**	**T**	**X**	sorted

4. Here is C++ code for quicksort. Assume that the global array `int A[n]` has been declared. The goal is to sort A.

```
int const n = 20;
int lo;
int hi;

int A[n];

void swap(int&x, int&y)
 {
  int z = x;
  x = y;
  y = z;
 }

void display()
 {
  for(int i = 0; i < n; i++)
  cout << " " << A[i];
  cout << endl;
 }

void quicksort(int first, int last)
 // sorts A[first..last]
 {
  assert(0 <= first and first <= last and last < n);
  if(first < last) // What happens if first = last?
   {
    int pivot;
    int mid = (first+last)/2;
    pivot = A[first];
    swap(A[first],A[mid]);
    pivot = A[first];
    lo = first;
    hi = last;
    while(lo < hi) // This is the main loop
     {
      if(A[hi] < A[lo+1])
       swap(A[hi],A[lo+1]);
      while(A[lo+1] < pivot) lo++;
      while(A[hi] > pivot) hi--;
     }
```

```
    assert(lo == hi);
    swap(A[first],A[lo]);
    quicksort(first,lo);
    quicksort(lo+1,last); // Oops!  There might be a bug here!
   }
 }

int main()
 {
  display();
  quicksort(0,n-1);
  display();
  return 1;
 }
```

The output of my program:

```
70 29 18 45 91 58 64 97 95 67 92 19 63 27 84 73 60 72 42 99
18 19 27 29 42 45 58 60 63 64 67 70 72 73 84 91 92 95 97 99
```

Answer the following questions.

(a) What is the loop invariant of the main loop?

lo $\leq$ hi and A[i] $\leq$ pivot for all first $\leq$ i $\leq$ lo and A[i] $>$ pivot for all hi $<$ i $\leq$ last.

(b) I was careful not to have any duplicate entries in my initial array, the first line of my output. Did I really need that condition for the program to work correctly? Explain.

If all entries from first to last are equal, the partition loop will split the array into one of size 1, the other with all the others. That causes quicksort to run in quadratic time.

If there are many duplicates, eventually there will be a recursive call to an array with identical entries.

(c) I treated the case where the array has length 2 with special code. Was that necessary?

(d) There might be a bug in the program, as indicated at the second recursive call. Can you find it? Even if there is a bug, the program might run correctly, but it might not.