University of Nevada, Las Vegas Computer Science 477/677 Spring 2025

Answers to Assignment 5: Due Saturday March 8 2025

For each shortest path problem, let n be the number of vertices, and m the number of arcs.

1. What is the worst case asymptotic time complexity for each of these shortest path algorithms?

   (a) Bellman Ford $O(nm)$

   (b) Floyd Warshall $\Theta(n^3)$

   (c) Dijkstra $O(m \log n)$

   (d) Johnson $O(nm \log n)$

3. Write pseudocode for the Bellmain Ford algorithm. Be sure to include the shortcut.

   Assume the vertices are $0, 1, \ldots n-1$, where 0 is the source vertex. Let $e_1, \ldots e_m$ be the arcs of G. The arc e_j is from s_j to t_t and has weight w_j.

   $V(0) = 0$
   For all i from 1 to $n-1$
   $V(i) = \infty$ For all j updatetime$(s_j) = 0$
   finished = false
   For all ℓ starting at 1 until finished, or $\ell = n$ or $V(0) < 0$
   finished = true
   For all j from 0 to m-1
   IF updatetime$(s_j) < \ell - 1$
   temp $= V(s_j) + w_j$
   $V(t_j) =$ temp
   back$(t_j) = s_j$
   backweight$(t_j) = w_j$
   updatetime$(t_j) = \ell$
   finished = false
   If $V(0) < 0$ or not finished Write "Error! There is a negative cycle."

2. Write pseudocode for the Floyd Warshall algorithm. Let W[i,j] be the length of the arc from i to j, which could be ∞.

   Assume that vertices are numbered $0 \ldots n-1$ For all i, j $V[i, j] = \infty$
   For each arc i, j $V[i, j] = W[i, j]$ and back$[i, j] = i$
   For all j from 0 to n
   For all i from 0 to n
   For all k from 0 to n
   temp $= V[i, k] + V[k, j]$
   If temp $< V[i, k]$ $[V[i, k] =$ temp
   back$[i, k] =$ back$[j, k]$

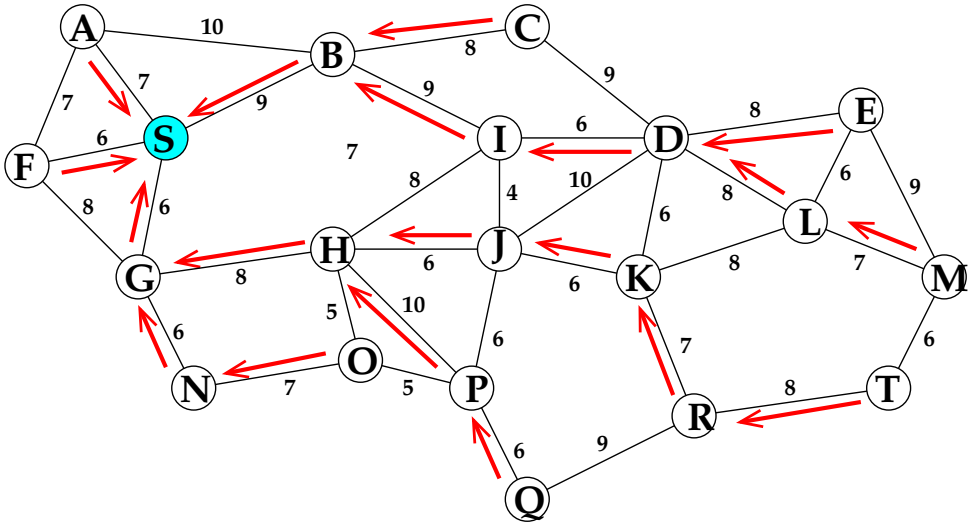
4. Walk through Dijkstra's algorithm for the single source minpath problem for the directed graph illustrated below. Instead of numbering the vertices 0 through 19, I have assigned them letters from A to T. The source vertex is S.

After each iteration of the main loop, show

1. The array dist, where dist[x] is the smallest length of any path found so far from S to x. (Initially, dist[x] = ∞ for most x.)

2. The array back, where back[x] is the next-to-the last vertex on the path of smallest weight found so far from S to x.

3 The contents of heap. Do not try to show the structure of the heap, simply list its members.



S	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	T
0	7	9	17	~~26~~ 24	32	6	6	14	18	20	26	34	39	12	19	24	30	33	41
*	S	S	B	~~C~~ I	D	S	S	G	B	H	J	K	L	G	H	H	P	K	R

S	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	T	minheap
0																				
*																				S0
0	7	9				6	6													deletemin S
*	S	S				S	S													F6 G6 A7 B9
0	7	9				6	6	14						12						deletemin F,G
*	S	S				S	S	G						G						A7 B9 N12 H14
0	7	9				6	6	14						12						deletemin A
*	S	S				S	S	G						G						B9 N12 H14
0	7	9	17			6	6	14	18					12	14					deletemin B,N
*	S	S	B			S	S	G	B					G	N					H14 C17 I18 O19
0	7	9	17	26		6	6	14	18	20				12	19	24				deletemin C,H
*	S	S	B	C		S	S	G	B	H				G	N	H				I18 O19 J20 P24 D26
0	7	9	17	24		6	6	14	18	20				12	19	24				deletemin I, O
*	S	S	B	I		S	S	G	B	H				G	N	H				J20 P24 D24
0	7	9	17	24		6	6	14	18	20	26			12	19	24				deletemin J
*	S	S	B	I		S	S	G	B	H	J			G	N	H				P24 D24 K26
0	7	9	17	24	32	6	6	14	18	20	26	32		12	19	24	30			deletemin P,D
*	S	S	B	I	D	S	S	G	B	H	J	D		G	N	H	P			K26 Q30 E32 L32
0	7	9	17	24	32	6	6	14	18	20	26	32		12	19	24	30	33		deletemin K
*	S	S	B	I	D	S	S	G	B	H	J	D		G	N	H	P	K		Q30 E32 L32 R33
0	7	9	17	24	32	6	6	14	18	20	26	32	39	12	19	24	30	33		deletemin Q,E,L
*	S	S	B	I	D	S	S	G	B	H	J	D	L	G	N	H	P	K		R33 M39
0	7	9	17	24	32	6	6	14	18	20	26	32	39	12	19	24	30	33	41	deletemin R
*	S	S	B	I	D	S	S	G	B	H	J	D	L	G	N	H	P	K	R	M39 T39
0	7	9	17	24	32	6	6	14	18	20	26	32	39	12	19	24	30	33	41	deletemin T
*	S	S	B	I	D	S	S	G	B	H	J	D	L	G	N	H	P	K	R	M39
0	7	9	17	24	32	6	6	14	18	20	26	32	39	12	19	24	30	33	41	deletemin M
*	S	S	B	I	D	S	S	G	B	H	J	D	L	G	N	H	P	K	R	empty

5. Write C++ code (which must be able to be executed) for a dynamic program which finds the maximum weight legal subsequence of a sequence of positive integers. A subsequence is legal if it has no consecutive terms of the input sequence. Your program should use backpointers to recover the best subsequence.

   In order to make the grader's task easier, here are three sequences, each of length 20. I want you to use while testing your program.

   15,324,184,48,102,31,119,15,26,160,129,78,14,7,64,83,22,88,185,123.

   178,101,150,192,175,152,80,148,37,28,173,57,111,63,123,159,167,183,116,34.

   139,79,19,53,52,23,94,298,209,59,16,116,80,130,279,195,6,37,161,206.

   Remember that the task of this assignment is not just to find the correct answers, but to write and run a correct program. My program is located at `coinrow.cpp` and the output of the program is:

```
15,324,184,48,102,31,119,15,26,160,129,78,14,7,64,83,22,88,185,123.
324 + 102 + 119 + 160 + 78 + 7 + 83 + 88 + 123 = 1084
```