A^* Algorithm

The A^* algorithm solves the single source minpath (least weight path) problem for a weighted directed graph. Let $w(x, y)$ be the weight of the arc, from x to y.

In the example below, the edges are not directional, but we simply assume that each edge represents two arcs, one in each direction.

A^* is an "intelligent" version of Dijkstra's algorithm, which finds the a minal path from the source S to all vertices. A^* is restricted to just one target vertex, T.

Heuristic

To work the algorithm, we must first obtain a value $h(x)$, shown in red in the figues, for each vertex x. For each x, $h(x)$ must be a positive number which is no greater than the least distance from x to T. Letting $h(x) = 0$ for all x is a valid choice: in that case the rounds of A^* duplicate the steps of Dijkstra's algorithm. The best choice is to let $h(x)$ be the true distance from x to T. That choice is clearly not obtainable in practice, since if we knew those values, we would already have a solution!

The heuristic should satisfy $h(x) - h(y) \geq w(x, y)$ for all vertices x and y. We say h is *monotone* or *consistent*.

Crow Flies

In an important practical case, where the distance from x to T follows a system of roads, a good choicd of heuristic could be the geodesic distance. Consistency is guaranteed in this case.

Steps of A^*

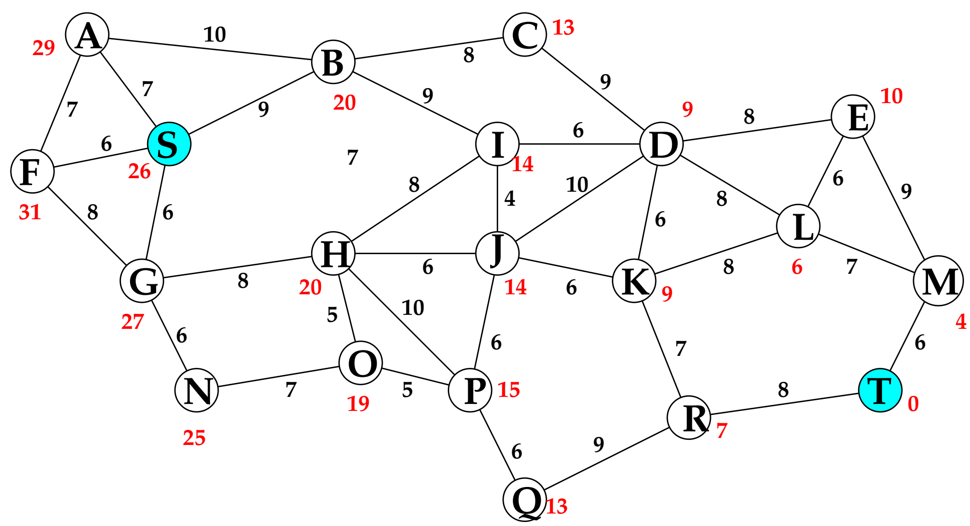
As in Dijkstra's algorithm, every vertex is either unprocessed, partially processed (OPEN), or fully processed (CLOSED) at each round. Inially, S is partially processed and all other vertices are unprocessed.

If x is partially processed, $f(x)$, shown in black in the figures, is the least cost of any path from S to x found so far. If x is fully processed, $f(x)$ is the least cost of any path from S to x.

For fully and partially processed vertices, $g(x) = f(x)+h(x)$, shown in green in the figures.

At each round of the A^* algorithm, the following steps are executed.

1. The partially processed vertex x which has the smallest value of $g(x)$ is chosen.
2. For each out-neighbor y of x which is unprocessed, let $f(y) = f(x) + w(x, y)$ y becomes partially processed. the back pointer back(x) = y shown as a dashed magenta arrow in the figures, is defined.
3. For each out-neighbor z of x which is partially processed. compute temp $= f(z) + w(x, z)$. If temp $< f(x)$, redefine $f(x) =$ temp and redefine back$(x) = z$.
4. x is now fully processed.
5. If $x = T$, the algorithm halts. The least cost path, of weight T, may be found by following back pointers starting at T.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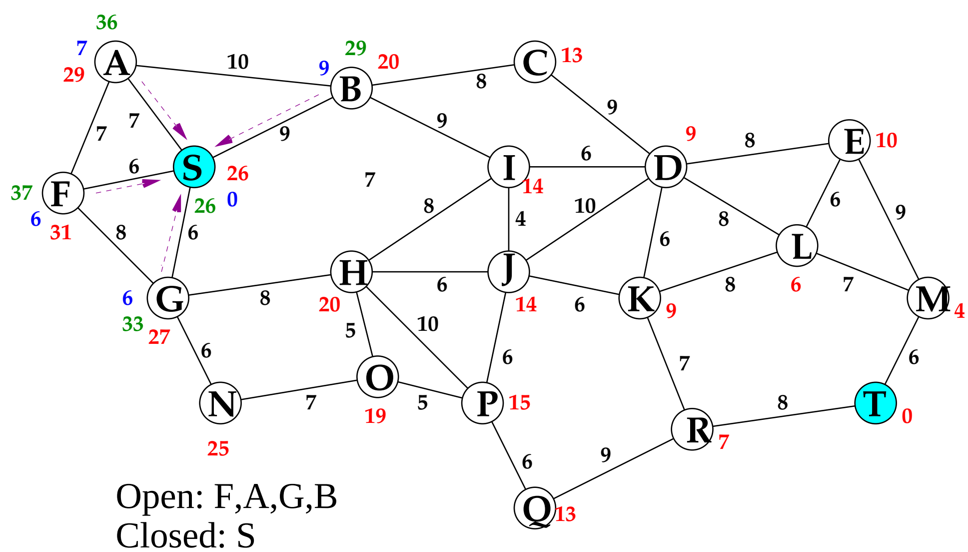
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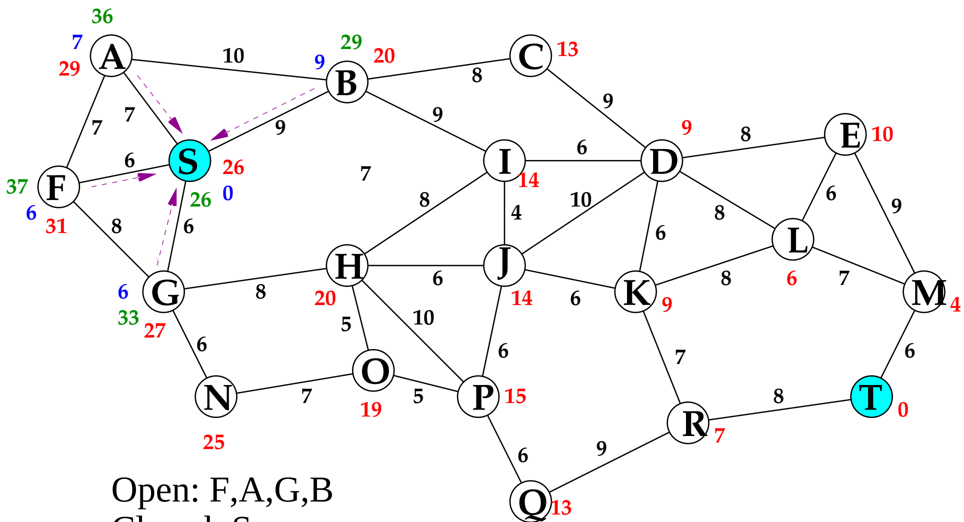

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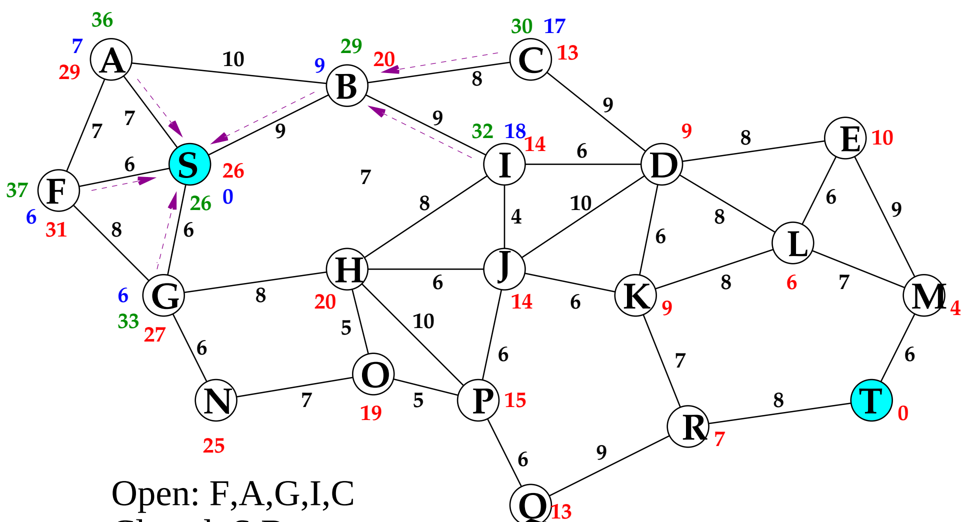

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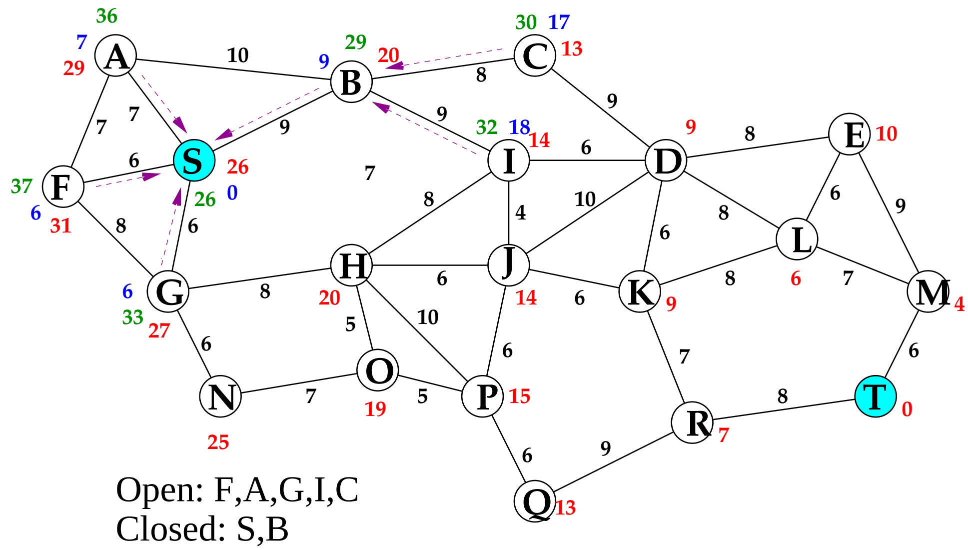
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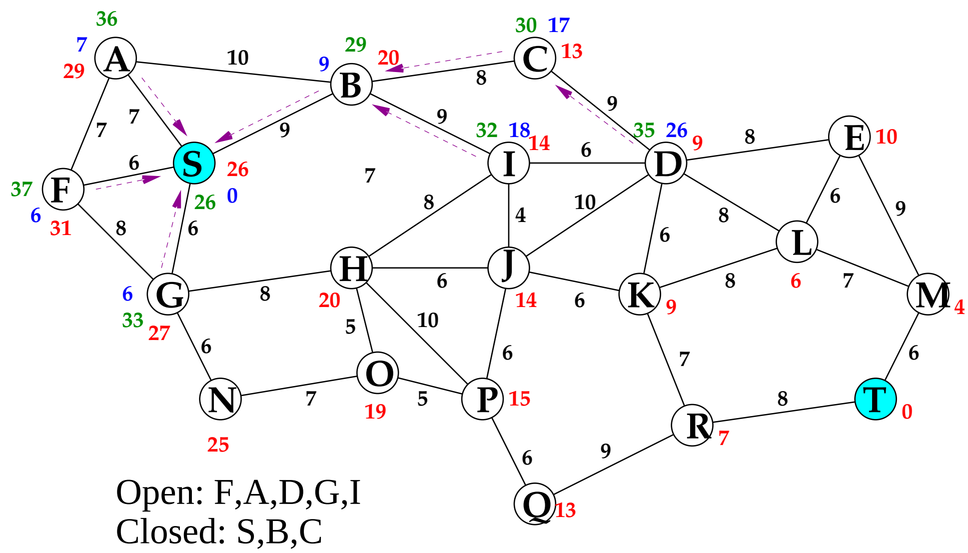
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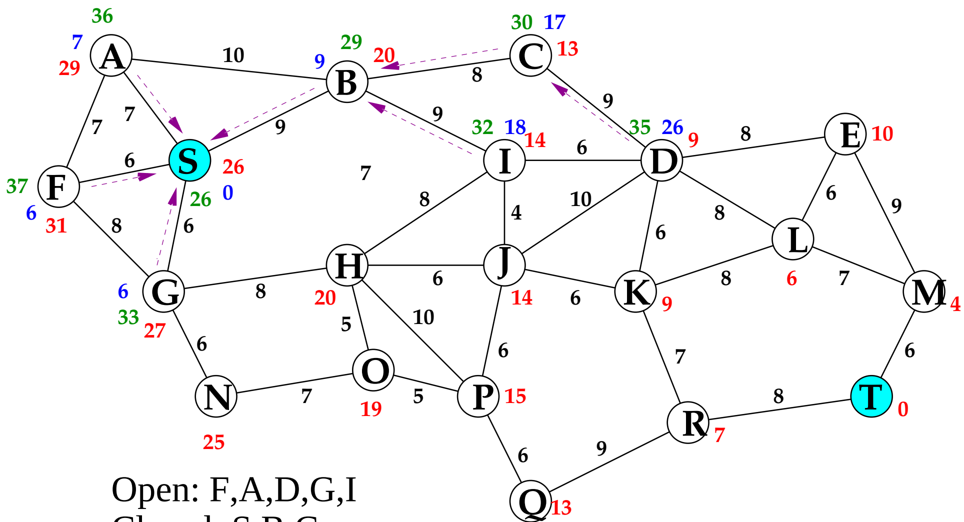

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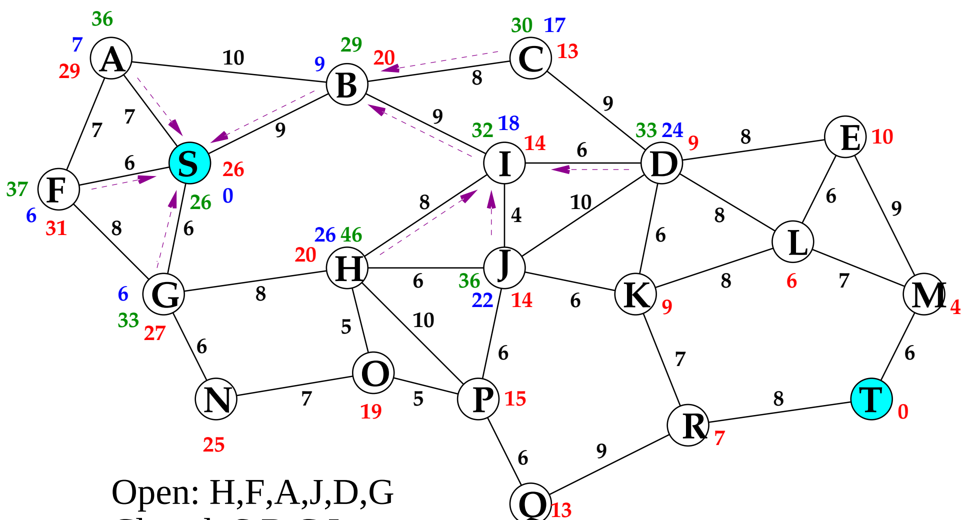
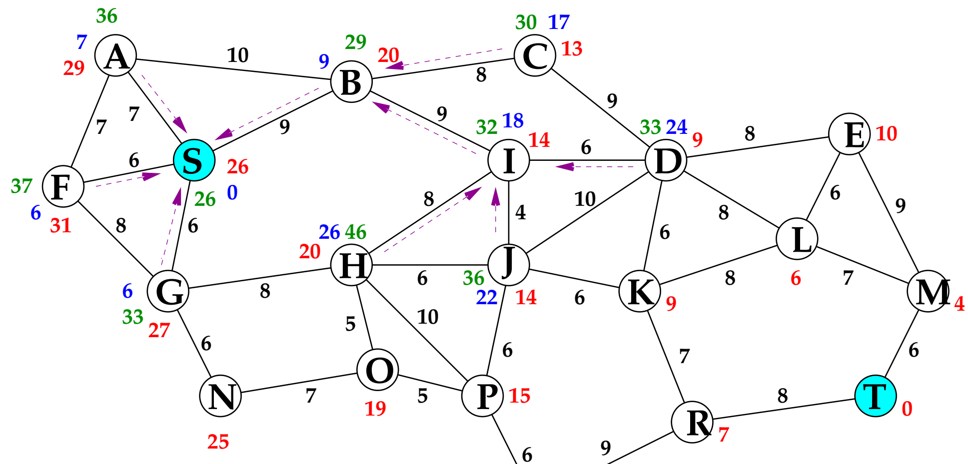


Open: H,F,A,J,D,G
Closed: S,B,C,I



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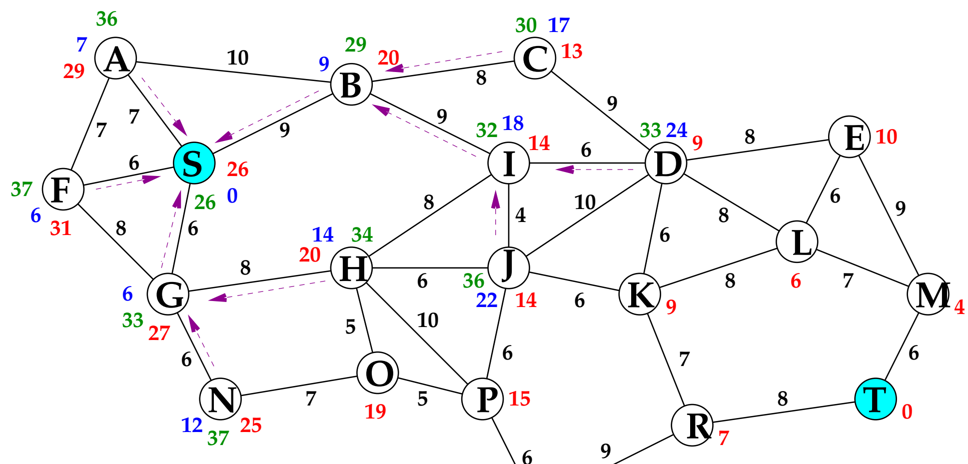

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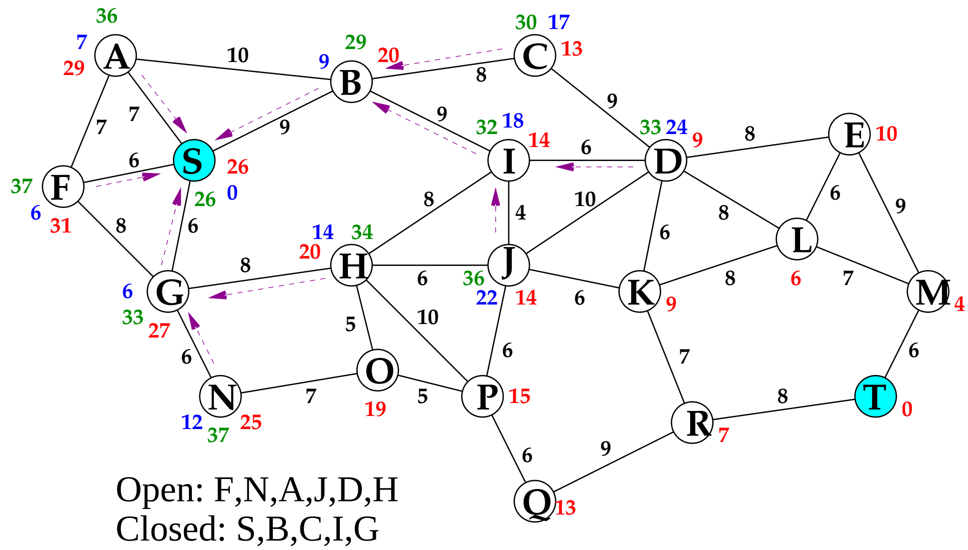
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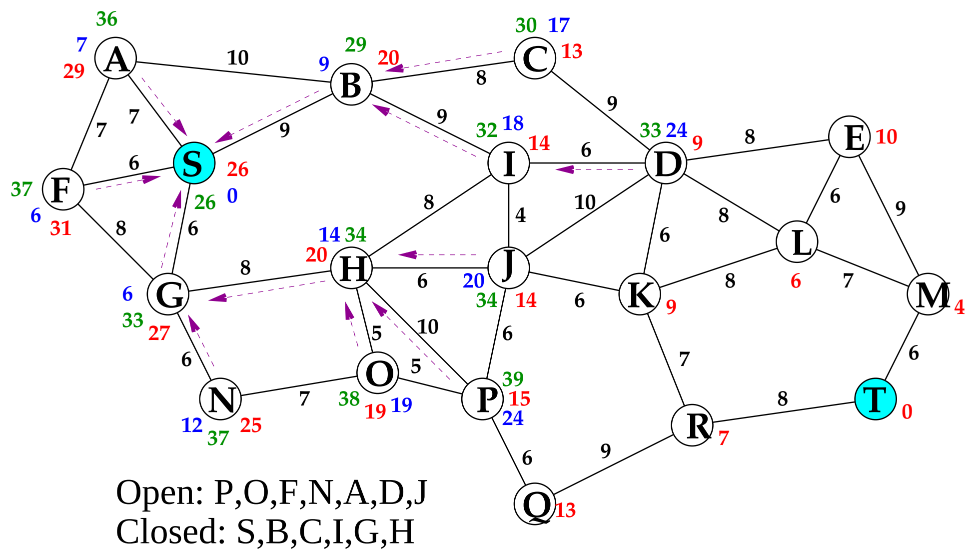
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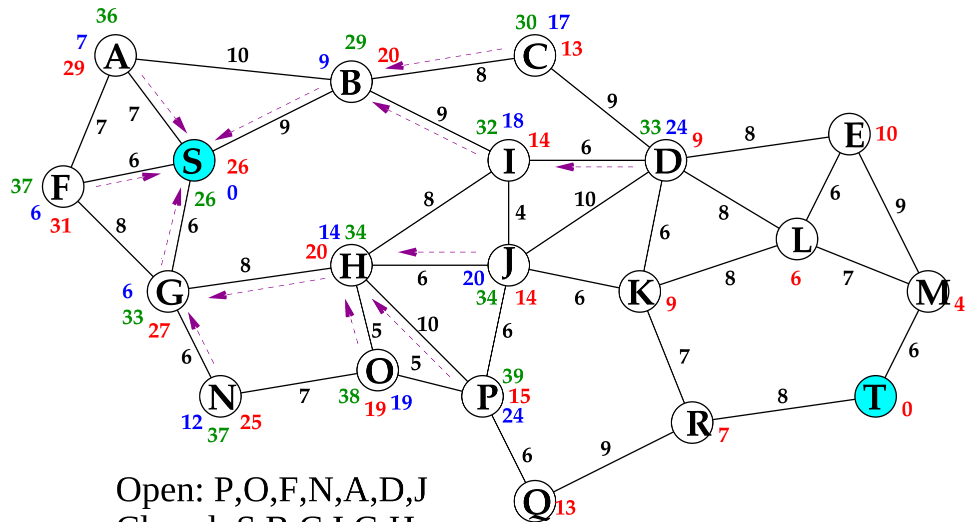

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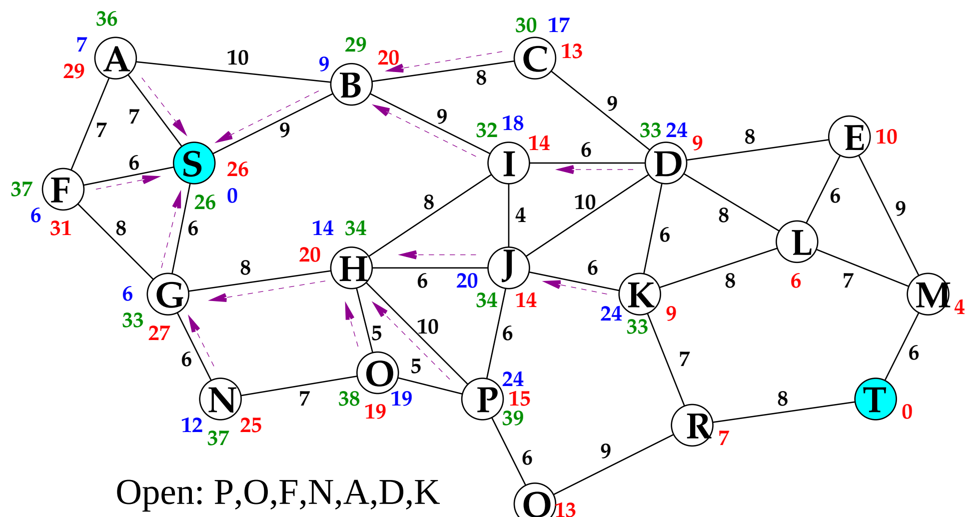

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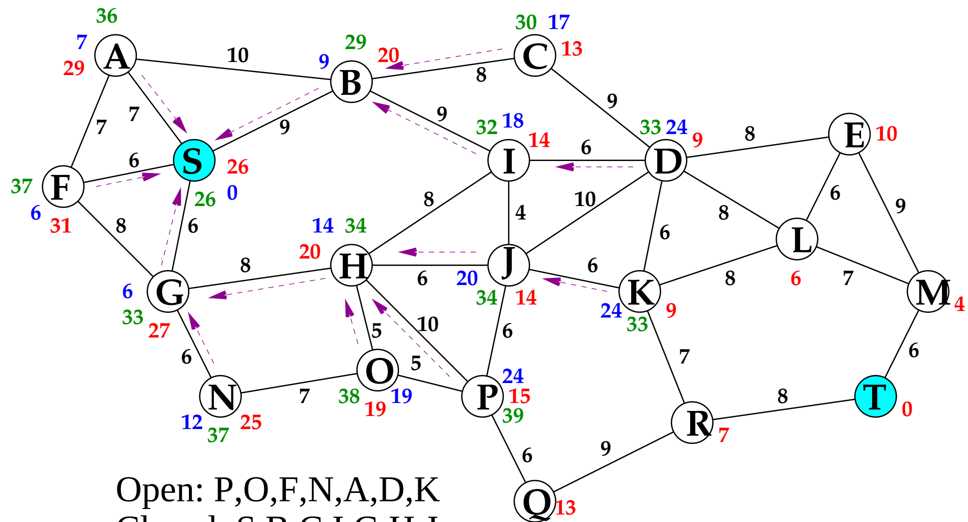

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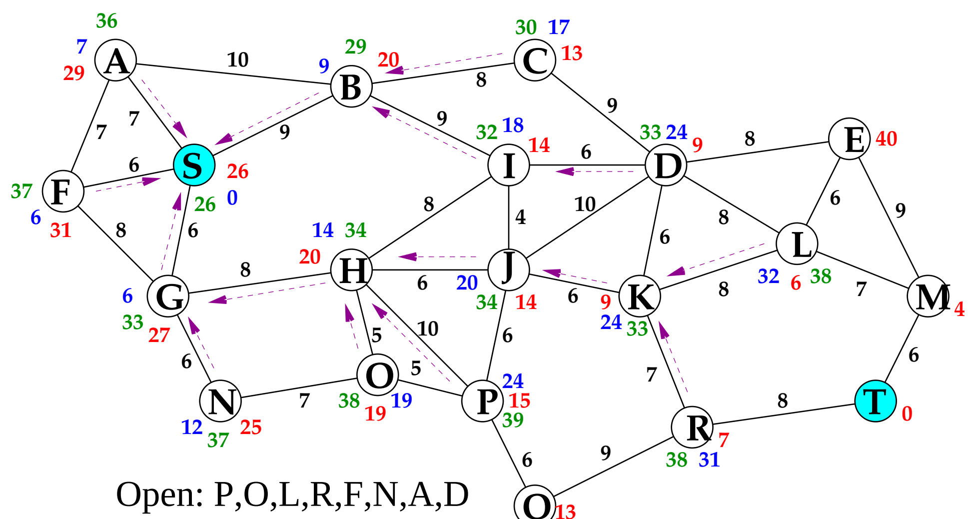

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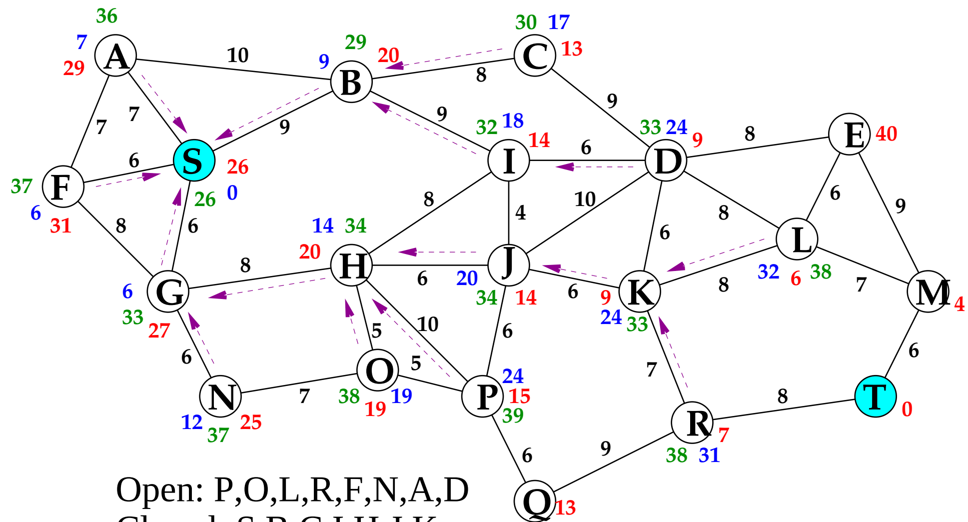

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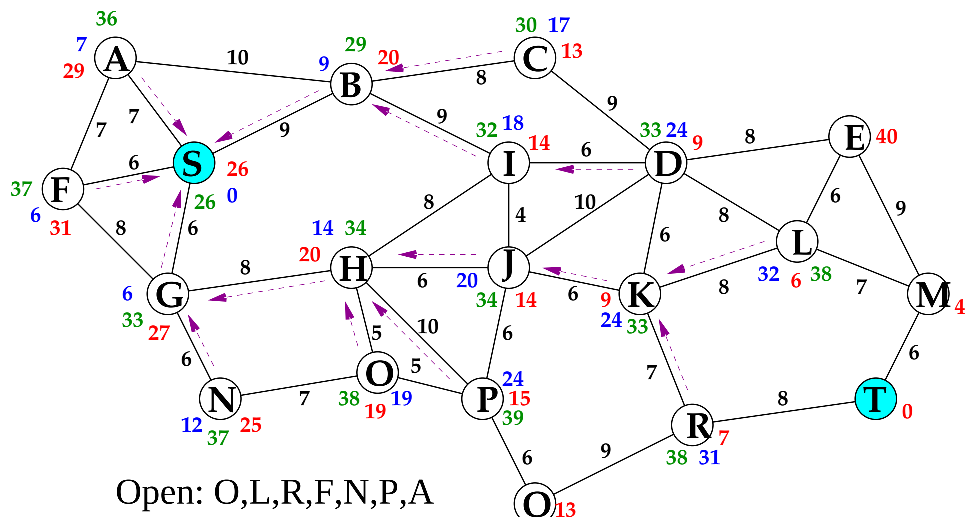

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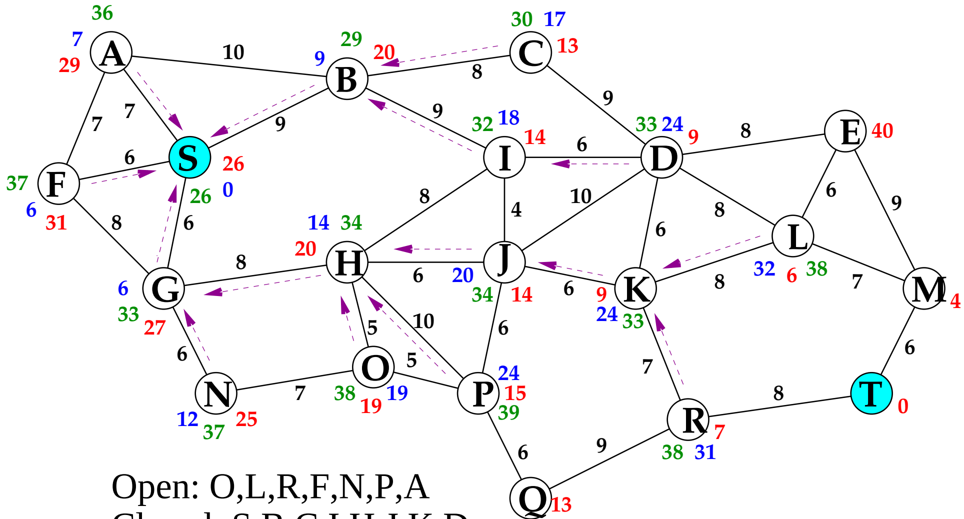

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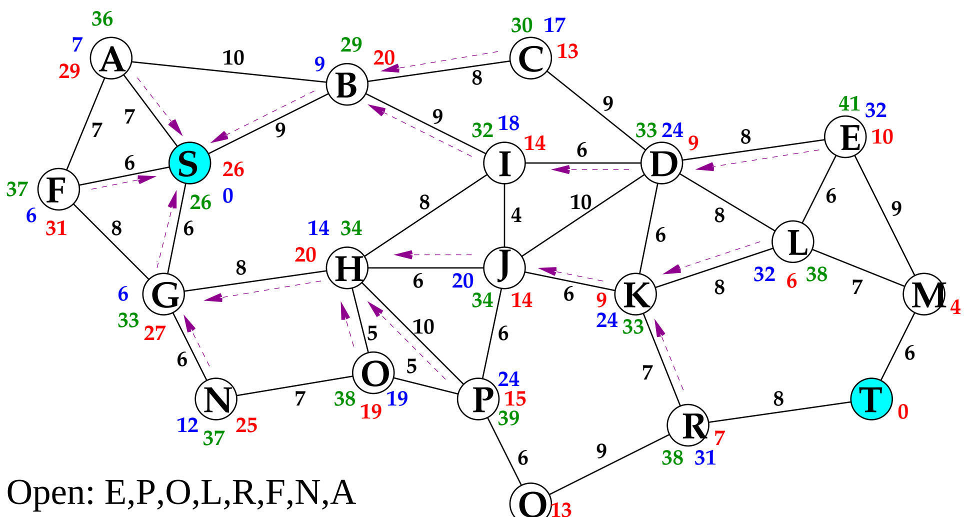

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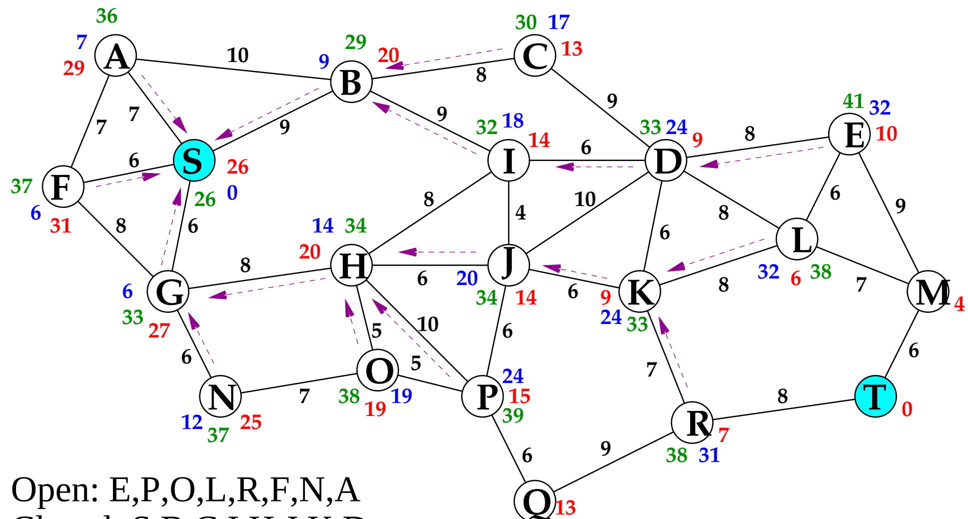

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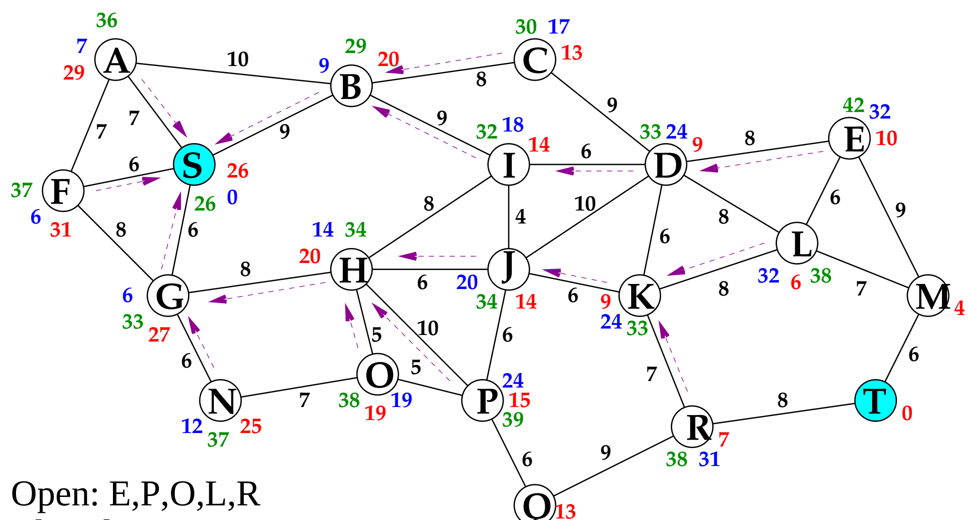

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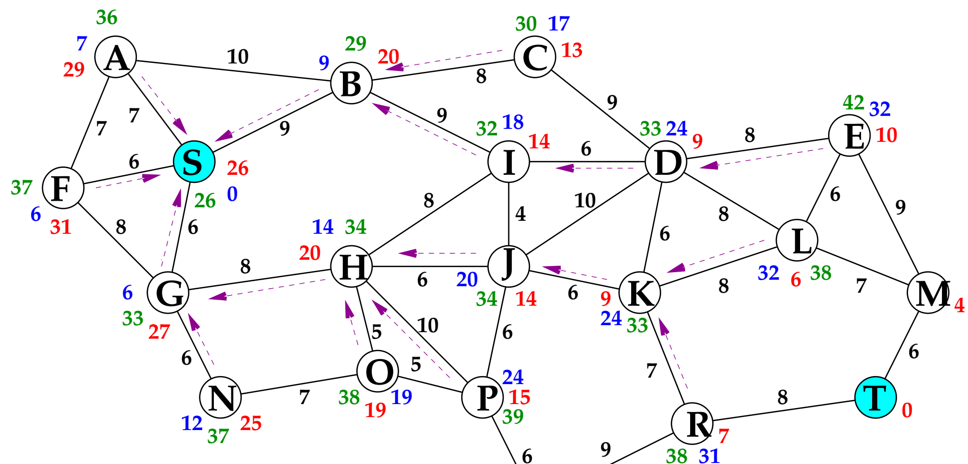

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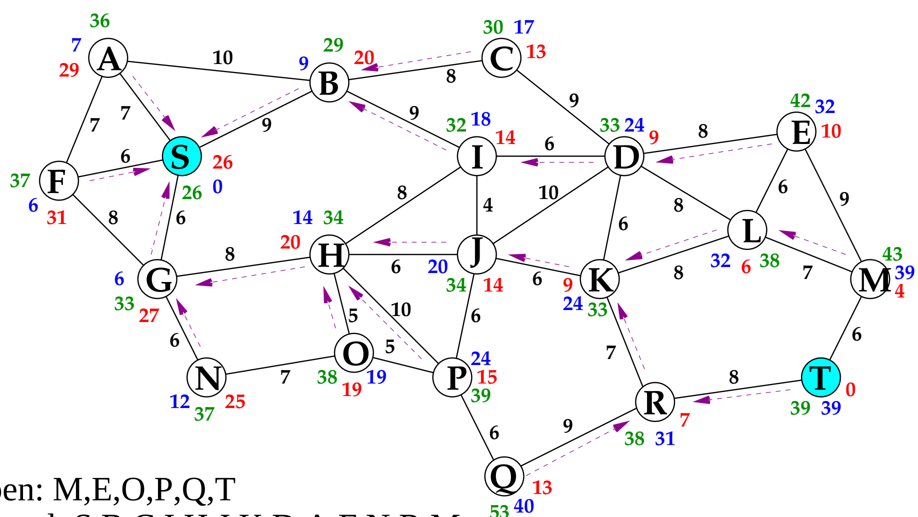

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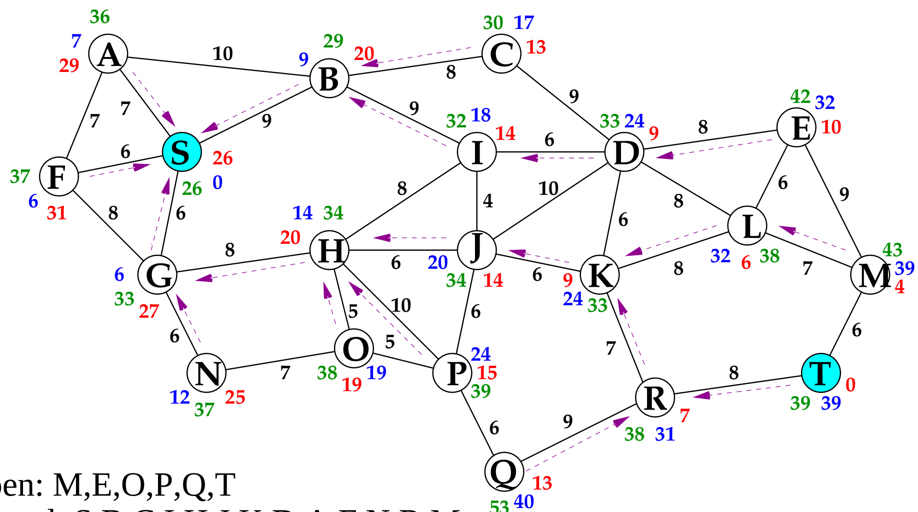

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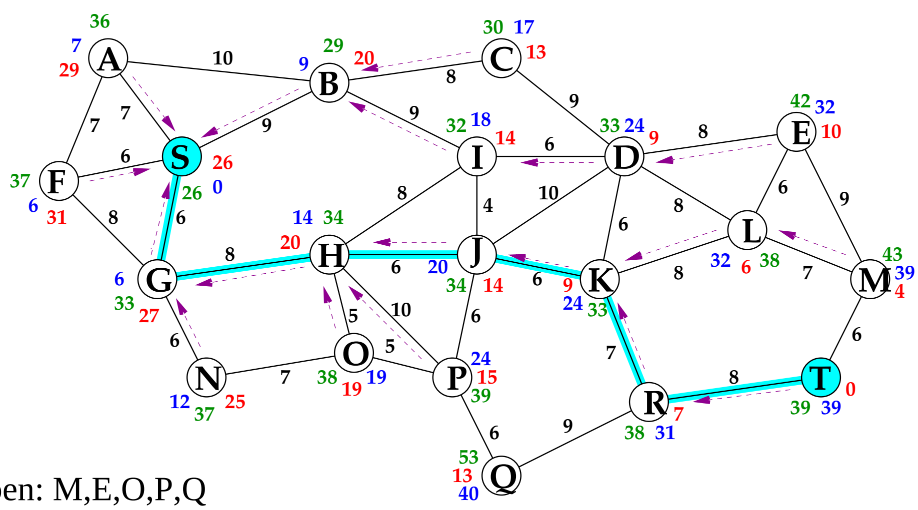

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