University of Nevada, Las Vegas Computer Science 477/677 Spring 2025

Answwers to Third Examination April 9, 2025

Name:____

The entire examination is 280 points.

1. True or False.

   (a) [5 points] **F** Open hashing uses open addressing.

   (b) [5 points] **F** Open hashing uses probe sequences.

   (c) [5 points] **F** You can avoid collisions in a hash table by making the table twice as large as the data set.

   (d) [5 points] **T** A hash function should appear to be random, but cannot actually be random.

2. Solve each of these recurrences, giving an asymptotic answer in terms of n, using Θ notation.

   (a) [5 points] $F(n) = 4F(n/2) + n$

   $F(n) = \Theta(n^2)$

   (b) [5 points] $F(n) = 3F(n/2) + n^2$

   $F(n) = \Theta(n^2)$

   (c) [5 points] $F(n) = F(n/2) + F((n-1)/2) + 3n$

   $F(n) = \Theta(n \log n)$

   (d) [5 points] $F(n) = 3F(n/3) + 3F(2n/3) + n^2$

   $F(n) = \Theta(n^3)$

   (e) [5 points] $F(n) = 2F(n/4) + \sqrt{n}$

   $F(n) = \Theta(\sqrt{n} \log n)$

   (f) [5 points] $F(n) = F(n/2) + 2F(n/4) + n$

   $F(n) = \Theta(n \log n)$

   (g) [5 points] $F(n) = F(n/3) + F(n/2) + n$

   $F(n) = \Theta(n)$

   (h) [5 points] $F(n) = F(n - \sqrt{n}) + n$

   $F(n) = \Theta(n^{3/2}) = \Theta(n\sqrt{n})$

(i) [5 points] $F(n) = F(n-1) + n^2$

$F(n) = \Theta(n^3)$

(j) [5 points] $F(n) = F(\sqrt{n}) + 1$

$F(n) = \Theta(\log\log n)$

(k) [5 points] $F(n) = 2F(3n/4) + F(n/2) + 2F(n/4) + 2n^3$

$F(n) = \Theta(n^3 \log n)$

(l) [5 points] $F(n) = 3F(n-1) + 1$

$F(n) = \Theta(3^n)$

(m) [5 points] $F(n) = F(\log n) + 1$

$F(n) = \Theta(\log^* n)$

3. Find the asymptotic time complexity of each code fragment, in terms of n. Use Θ if possible.

   (a) [10 points]

```
for(int i = 1; i < n; i++)
 for(int j = 1; j < i; j++)
\vskip 0.1in
$\Theta(n^\maly2)$
\vskip 0.1in
```

   (b) [10 points]

```
for(int i = 1; i < n; i++)
 for(int j = i; j < n; j++)
```

$\Theta(n^2)$

   (c) [10 points]

```
for(int i = 1; i < n; i = 2*i)
  for(int j = 2; j < i; j = j*j)
```

$F(n) = \Theta(\log n \log\log n)$

4. Fill in the blanks.

   (a) [5 points] **perfect** hashing, which can be used by a compiler to identify reserved words, does not have collisions.

   (b) [5 points] In closed hashing, if a collision occurs, a **probe** sequence can be used to locate an unused position in the hash table.

   (c) [5 points] In a **cuckoo** hash table, each item has two or more possible locations, and must be stored in one of those.

(d) [5 points] In an open hash table, there is a **search structure** at each table index.

(e) [10 points] A directed graph is defined to be **strongly connected** if, given any two vertices x and y, the graph contains a path from x to y.

(f) [5 points] In order for there to exist a topological order of the vertices of a digraph, the graph must be **acyclic**

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5. [20 points] A 3-dimensional $8 \times 9 \times 6$ rectangular array X is stored in main memory in column major order, and its base address is 4096. Each item of X takes two words of main memory, that is, two address location. Find the address, in main memory, of X[3][7][4].

The number of column-major predecessors of X[i][j][k] in X[N][M][O] is

$M * N * k + N * j + i$, which is $8 * 9 * 4 + 8 * 7 + 3 = 288 + 56 + 3 = 347$ in this example. The address of the item in main memory is $4096 + 2 * 374 = 4844$.

6. [20 points] You are trying to construct a cuckoo hash table of size 10, where each of the 10 names listed below has the two possible hash values indicated in the array. Put the items into the table, if possible; otherwise, convince me it's impossible. Instead of erasing ejected items, simply cross them out, so that I can tell that you worked it properly.

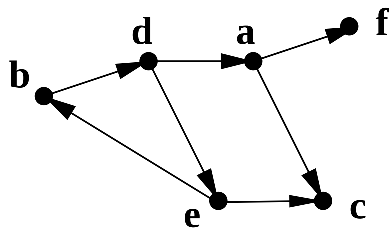
	h1	h2
Ann	5	9
Ben	0	8
Cal	4	6
Deb	1	8
Eve	1	0
Fay	9	4
Gus	7	2
Hal	3	7
Ike	3	2
Jan	1	7

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

There is no solution. The graph theoretic formlation of Hall's marriage theorem can be used to determine whether a solution exists. That theorem states that there is a solution if and only if, for any set A of n data, the combined set of hash values of members of A must have at least n elements. In our case, there is no solution because there is a set of 7 data which together have only 6 hash values.

If you follow the algorithm I showed you, you will never find a solution, eventually entering an endless cycle.

7. [20 points] Write the in-neighbor list and out-neighbor list representations of the directed graph shown below.



a	d
b	e
c	a, e
d	b
e	d
f	a

a	c, f
b	d
c	
d	a, e
e	c
f	

8. [20 points] Let $\sigma = x_1, x_2, \ldots x_n$ be a sequence of numbers with both positive and negative terms. Write an $O(n)$ time dynamic program which finds the maximum sum of any contiguous subsequence of σ. For example, if the sequence is $-1, 4, -3, 2, 7, -5, 3, 4, -8, +6$ then the answer is $4-3+2+7-5+3+4 = 12$.

   Define $X[i]$ to be the maximum sum taken from the first i terms of the seqence. Define $Y[i]$ to be the maximum sum of any subsequence ending at x_i.

   $X[0] = 0$
   $Y[1] = x_1$
   $X[1] = max\{0, x_1\}$
   For i from 2 to n
   {
   $Y[i] = x_i + max\{0, Y_{i-1}\}$
   $X[i] = max\{Y[i], X[i-1]\}$
   }

   The answer is $X[n]$.

9. [20 points] Explain how to implement a sparse array using a search structure.

   Let A be the sparse virtual array. Let S be a search structure which contains ordered pairs of the form (i, x), where $A[i] = x$. To fetch the value of $A[i]$, search S for a pair (i, x). If that pair is found, return x, otherwise return a default value, such as 0. To store a value x for $A[i]$, search S for a pair (i, y). If that pair is found, replace y by x. That pair is not found, insert the pair (i, x) into S.

10. [10 points] Compute the Levenshtein edit distance from the word "proven" to the word "shore." Show the matrix.

		s	h	o	r	e
	0	1	2	3	4	5
p	1	1	2	3	4	5
r	2	2	2	3	3	4
o	3	3	3	2	3	4
v	4	4	4	3	3	4
e	5	5	5	4	4	3
n	6	6	5	5	5	4

The edit distance is 4.

11. [20 points] Write pseudocode for the simple coin-row problem. There are n coins in a row, each having some positive value. You with to select the set of coins of maximum value, but are not allowed to select any two coins which are adjacent.

    Let $W[i]$ be the maximum weight of any legal subset of the first i coins.

    $W[0] = 0$

    $W[1] = c_1$

    for(i = 2; i ¡= n; i++)

    $W[i] = max(A[i-1], c_i + W[i-2])$

    The answer is $W[n]$.

12. You are implementing a 3D triangular array A where $A[i][j][k]$ is

defined for $0 \leq k \leq j \leq i \leq 4$, a total of 35 entries (Is that correct?), and is stored as a one-dimensional subarray of main memory in row-major order, with base address 1024. Each term of A takes one place in main memory. What would be the address, in main memory, of $A[4][2][1]$?

If $i \geq j \geq k$, the number of predecessors of $A[i][j][k]$ is $\binom{i+2}{3} + \binom{j+1}{2} + \binom{k}{1}$ Thus, the number of predecessors of $A[4][2][1]$ is $\binom{6}{3} + \binom{3}{2} + \binom{1}{1} = 20 + 3 + 1 = 24$. Its address in main memory is $1024 + 24 = 1048$.

The formula can be extended to higher dimensions. In a 4 dimensional triangular array, the number of predecessors of $A[i][j][k][\ell]$, for $i \geq j \geq k \geq \ell$, is $\binom{i+3}{4} + \binom{j+2}{3} + \binom{k+1}{2} + \binom{\ell}{1}$.