

Assignment 4: Due Saturday April 4, 2026 11:59:59 PM

Name: _____

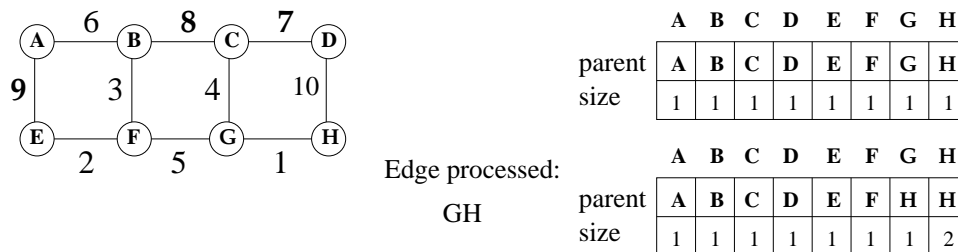
Advice of the Day: "When all else fails, follow instructions."

- On March 17, our graduate assistant, Rakibur Hassan, presented a complete explanation of how to use union/find to implement Kruskal's algorithm for finding a minimum weight spanning tree for a connected weighted graph.

Use union/find to implement Kruskal's algorithm for the weighted graph below. A step consists of deciding whether a given edge should be part of the minimum spanning tree. The first step considers the edge GH. $\text{find}(G) = G$, $\text{find}(H) = H$, and $\text{union}(G,H)$ (arbitrarily) moves G into the set led by H.

The first array shows the initial values of *parent* and *size* of each vertex. In the second array, $\text{parent}(G)$ has been changed to H, and $\text{size}(H)$ to 2.

Finish the execution of Kruskal's algorithm. At each step, show the matrix of parent and size values. Attach extra sheets as needed.



- Walk through the A^* algorithm to compute a minimum cost path from S to T in the weighted directed graph shown below. The heuristic h is shown in red, while the weight on each arc is shown in black.

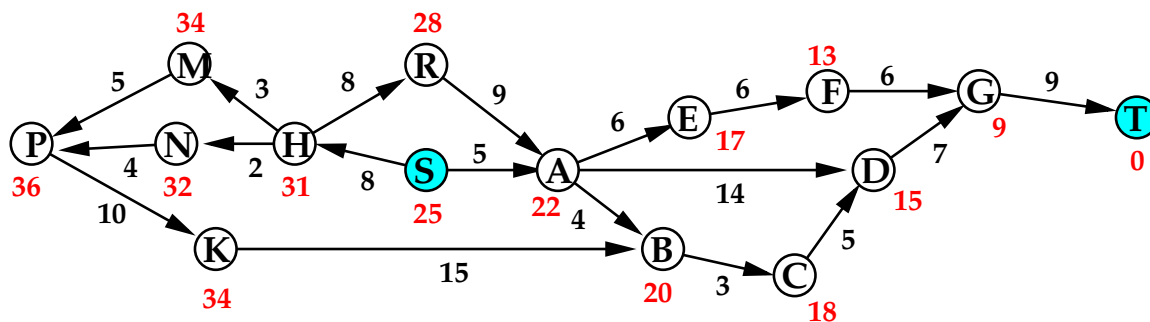
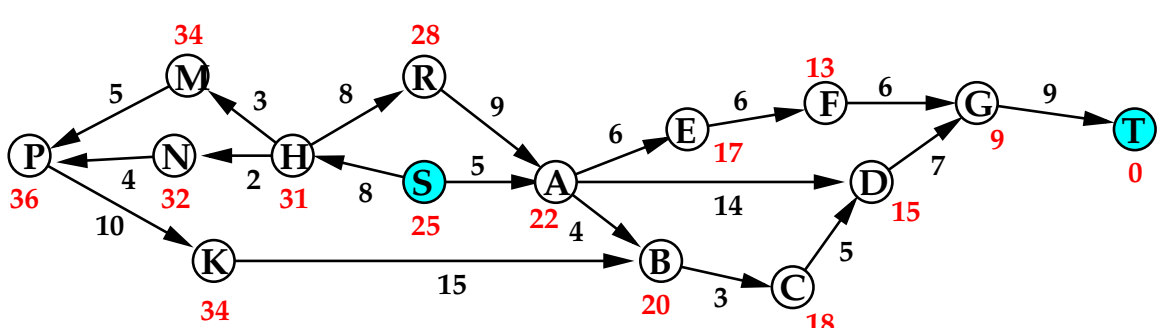
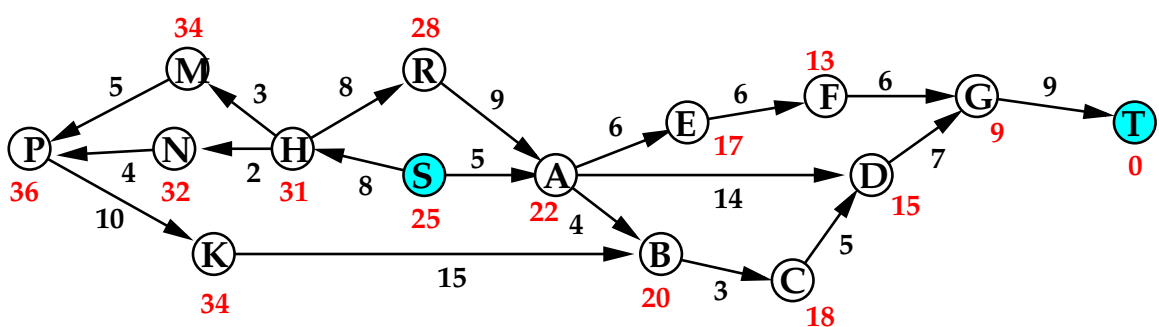
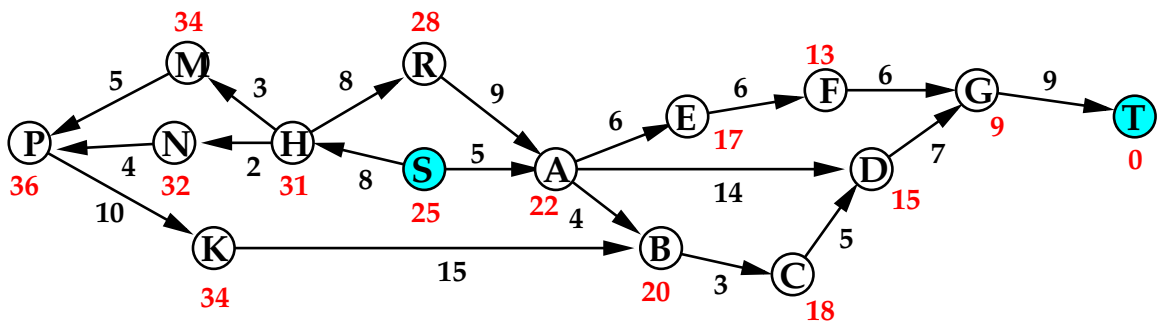
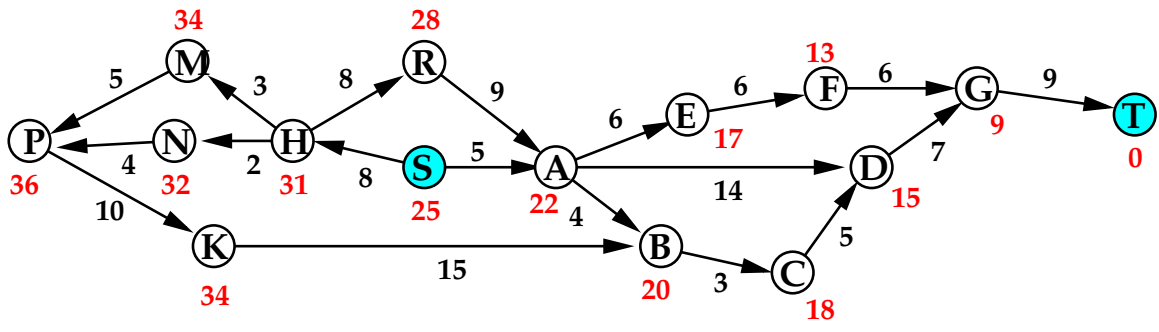
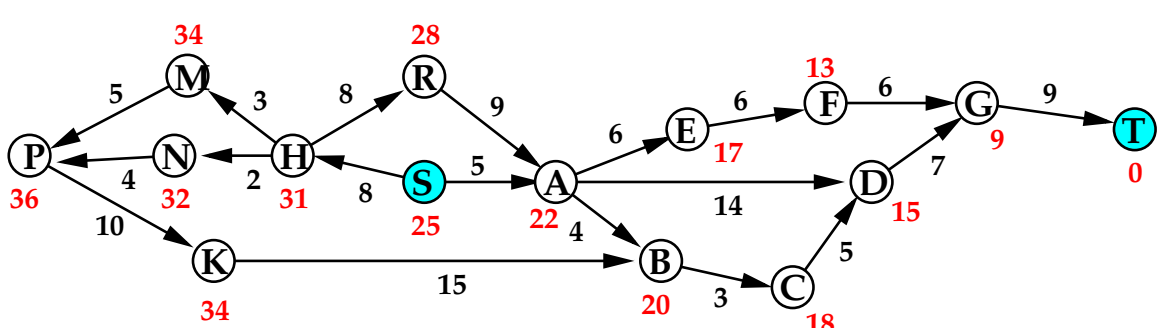
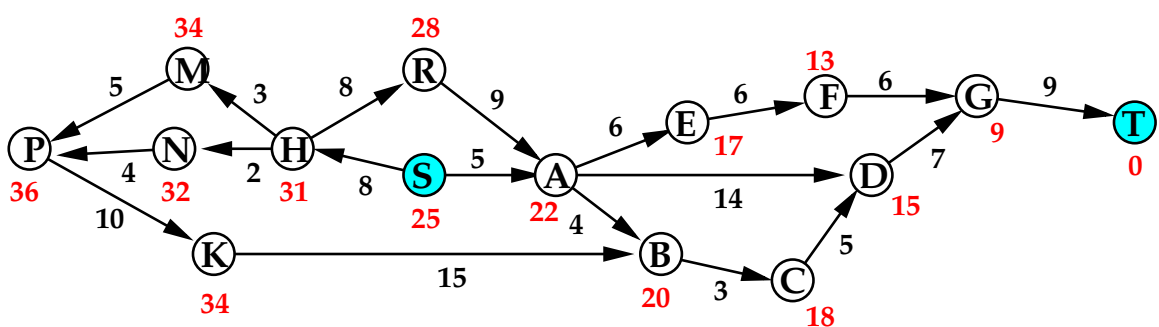
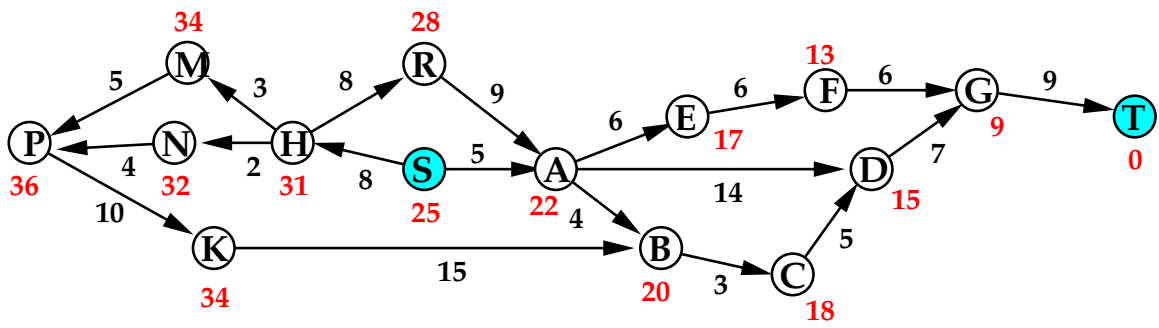
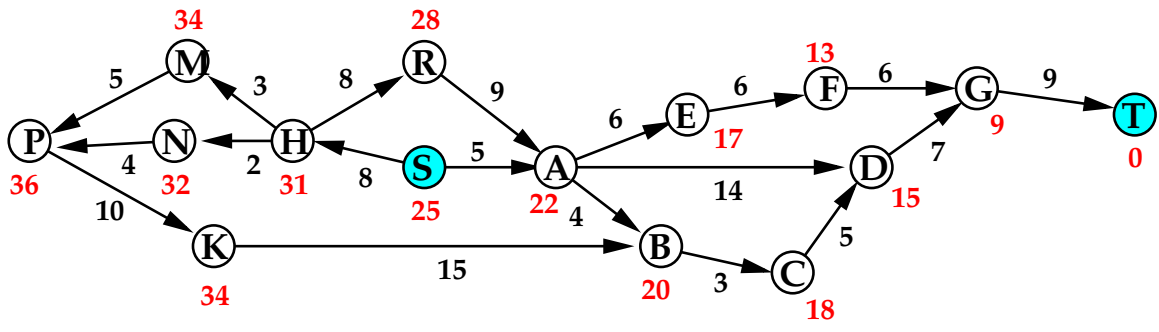


Figure 1: Example single pair minpath problem.

To avoid making you repeatedly redraw the figure, I have included several copies of Figure 1 on subsequent pages. If you need more of these figures, simply print another copy of one of those pages.





3. Consider the following definition of a function g .

As I announced in class, I have changed the recurrence for g so as to make the answers to the problem easier to calculate. In fact, without that change, there is no solution to part (c).

$$g(n) = 1 \text{ if } n \leq 4$$

$$g(n) = g((n-1)/2) + g(n/2) + g((n+1)/2) + g((n+2)/2) + n \text{ for } n > 4.$$

(a) The following recursive function computes $g(n)$.

```
int g(int n)
{
    if(n <= 4) return 1;
    else return g((n-1)/2)+g(n/2)+g((n+1)/2)+g((n+2)/2)+n;
}
```

What is the asymptotic time complexity, in terms of n , of that code?

(b) Here is a dynamic program which computes values of $g(n)$ for $0 \leq n \leq N$. Let G be an array.

```
for(int n = 0; n <= 4; n++)
    G[n] = 1;
for(int n = 5; n <= N; n++)
    G[n] = G[(n-1)/2] + G[n/2] + G[(n+1)/2] + G[(n+2)/2] + n;
```

What is the asymptotic time complexity, in terms of N , of that code?

(c) Use memoization to design an algorithm which computes $g(N)$ in $O(\log N)$ time, not counting the time needed for fetching memos from and storing memos into the search structure.