Computer Science 789 Randomized Algorithms Fall 2004 Examination October 28, 2004

No books, notes, scratch paper, or calculators. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided.

1. Consider the following process.

   (a) Initially, the state is 0.

   (b)

   (c)

   (d)

   What is the expected waiting time (*i.e.*, number of steps) to reach state 2?

2. Let $L = \{000, 010, 001, 100\}$.

   (a) Design a radomized algorithm to determine whether a given string w is in L. The cost of your algorithm is the number of bits of w you query. (Thus, your cost cannot exceed 3.)

   (b) Prove that your algorithm has expected cost less than 3.

   (c) Prove that no deterministic algorithm for the same problem can have cost less than 3.

3. Prove that there is a unique treap for any set of items, if the key values are distinct and the priorities are distinct.

4. Gamblers Ruin. Suppose that gamblers A and B play a game where at each step, A pays B 1 with probability $\frac{1}{2}$, otherwise B pays A 1. Initially, A has a and B has b, both positive integers. The game ends when one or the other player is ruined, *i.e.*, has nothing left.

   Prove that the expected waiting time for the game to end, *i.e.*, the expected number of steps, is finite.