Clues for the Underemployed Watchman

Clues for the Oblivious Forgetful Adversary

As we know, there is no online algorithm for computing the optimal service of a request sequence for the Underemployed Watchman problem. However, there is an online algorithm for computing the optimal cost of a request sequence.[1]

Each position (left or right) is marked in one of three ways: Y(es), N(o), or M(aybe). Initially, the left is marked Y, the right is marked N, and $cost_{\text{OPT}} = 0$.

> Mark the left side Y and the right side N.
>
> Let $cost_{\text{OPT}} = 0$.
>
> For each request, execute the following:
>
> > If the requested side is marked Y, do nothing.
> >
> > Else if the requested side is marked M, mark that side Y and the other side N.
> >
> > Else if the requested side is marked N, mark both sides M and increment $cost_{\text{OPT}}$ by 1.

The final value of $cost_{\text{OPT}}$ is the optimum cost of serving the sequence.

Let p^t be the probability that the marks are MM after t requests, if the request sequence is generated by the oblivious forgetful adversary. Then $p^0 = 0$, and $p^t = \frac{1}{2}\left(1 - p^{t-1}\right)$ for all $t > 0$. (Do you see why)? Thus, you can compute p^t for any $t \geq 0$, although the fractions get more and more complex as t increases.

However, the sequence $\left\{p^t\right\}$ converges to a simple fraction. What is that fraction? You can then compute the limiting value of the average expected optimum cost, namely:

$$\lim_{n \to \infty} \frac{E(cost^n_{\text{OPT}})}{n}$$

[1] The same is true for the paging problem.