8 Randomized Online Algorithms for the Server Problem

8.1 HARMONIC

HARMONIC is a lazy memoryless randomized algorithm for the k-server problem. Its competitiveness is not very good, but it is interesting because it is so simple.

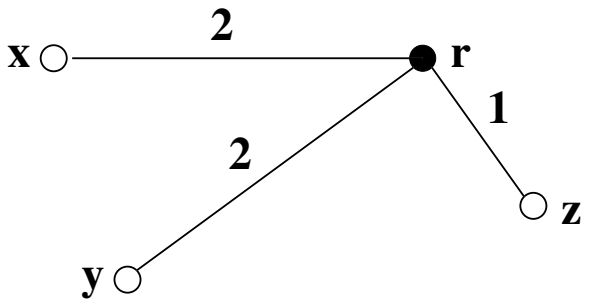


Figure 8.1: HARMONIC for $k = 3$: x and y each move with probability $\frac{1}{4}$, z moves with probability $\frac{1}{2}$.

Suppose that our servers are located at positions $\{x_i, \ldots, x_k\}$, and the request point is r. Let $d_i = d(x_i, r)$.

- If $s_i = r$ for some i, then HARMONIC moves no servers.
- Otherwise, for each i, HARMONIC moves the server from s_i with probability

$$p_i = \frac{d_i^{-1}}{d_1^{-1} + \cdots + d_k^{-1}}$$

Consider the example shown in Figure 7.1, where $k = 3$, $d_1 = x$, $d_2 = y$, and $d_3 = z$. Then $d_1^{-1} = \frac{1}{2}$, $d_2^{-1} = \frac{1}{2}$, $d_3^{-1} = 1$, and thus $d_1^{-1} + d_2^{-1} + d_3^{-1} = 2$. The probability that we move each server is then:

$$\begin{aligned} p_x = p_1 &= \frac{\frac{1}{2}}{2} = \frac{1}{4} \\ p_y = p_2 &= \frac{\frac{1}{2}}{2} = \frac{1}{4} \\ p_z = p_3 &= \frac{1}{2} \end{aligned}$$

Theorem 8.1 *The competitiveness of HARMONIC for the k-server problem is at least $\binom{k}{2} = \frac{k\,(k+1)}{2}$.*

Conjecture 8.2 *The competitiveness of HARMONIC for the k-server problem is $\binom{k}{2} = \frac{k\,(k+1)}{2}$.*

Theorem 8.3 *The competitiveness of HARMONIC for the 2-server problem is $\binom{3}{2} = 3$.*

Theorem 8.4 *HARMONIC is competitive for any k.*