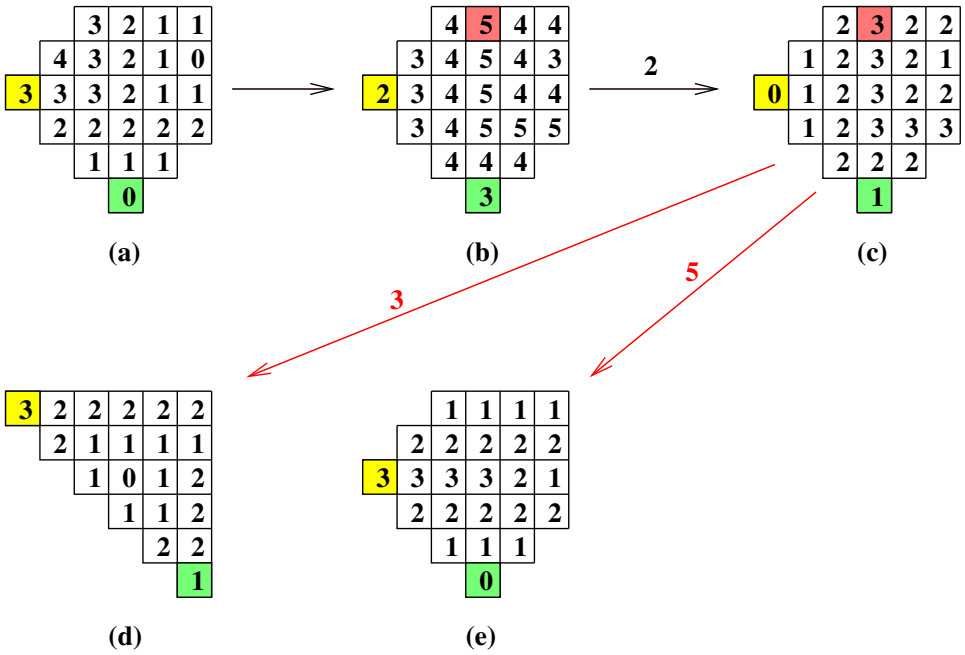


Figure 12.1: Example in $\mathcal{M}_5$

In Figure 12.1, we show an example of one step in the case that $M \in \mathcal{M}_5$. Initially, our two servers are at points r and s, such that $||r, s|| = 3$. We initially assume that the trackless work function is supported by the pair $\{r, s\}$ and the pairs $\{r, a\}$, where $||a, r|| = 5$ and $||a, s|| = 4$. That work function is shown in 12.1(a).

The next request is at a point r' such that $||r, r'|| = 3$ and $||s, r'|| = 5$. The resulting trackless work function is shown in 12.1(b). We offset by 2, and the result is shown in 12.1(c).

Figure 12.1(d) shows the trackless work function if we serve the request by moving our server from r to r'. Figure 12.1(e) shows the trackless work function if we serve the request by moving our server from s to r'.

Even metric spaces. We say that a metric space is *even* if all distances are integers, and the perimeter of any triangle is even. For example, a hypercube is an even metric space.

Let $\mathcal{E}$ be the class of all even metric spaces. For any d, let $\mathcal{E}_d = \mathcal{E} \cap \mathcal{M}_d$, and for any d and p, let $\mathcal{E}_{d,p} = \mathcal{E} \cap \mathcal{M}_{d,p}$.

Consider the class $\mathcal{E}_{3,6}$. The 3-dimensional cube H^3 is in that class.

Exercise 12.1 *Is there any member of $\mathcal{E}_{3,6}$ that has more than eight points?*

When we illustrate a trackless work function for the 2-server problem on M, where M is an even space, then half the values don't exist.

For example, Figure 12.2 shows a trackless work function for the class $\mathcal{E}_5$. Simply by not filling in squares which cannot correspond to points, we get (a). On the other hand, why not draw it as (b)?

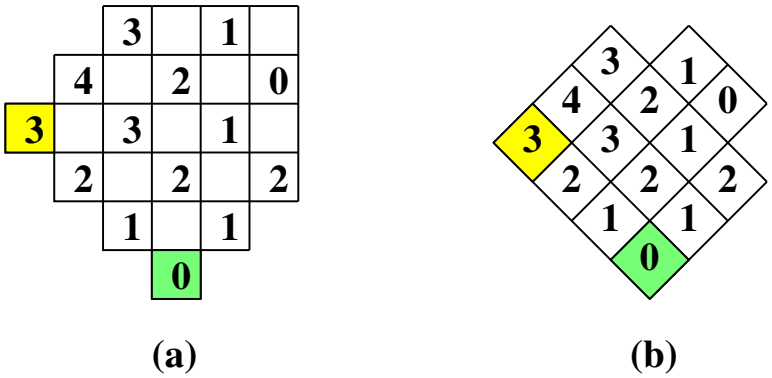


Figure 12.2: A Trackless work function for $\mathcal{E}_5$

Exercise 12.2 *What is the trackless competitiveness of the 2-server problem on $\mathcal{E}_{3,6}$?*

You know that the answer to Exercise 12.2 is at least 2, but no more than $\frac{11}{5}$. You could use a computer, but the problem is small enough to work out by hand.

The way to do it is to construct a red-black graph. You might want to make use of the following lemma.

Lemma 12.1 *If τ is a trackless work function for the 2-server problem on an even metric space, and if X and Y are pairs of points such that $||X, Y|| = 1$, then $|\tau(X) - \tau(Y)| = 1$.*

By Lemma 12.1, if you draw work functions in the same way as in Figure 12.2(b), any two adjacent numbers will differ by 1. My preliminary calculations indicate that there are 24 black nodes, three of which are cones. Possibly, the number of black nodes reachable from cones is less than 24. The number of red nodes should be similar.