Computer Science 789 Spring 2011 Final Examination, May 10, 2011

Name:_____

No books, notes, scratch paper, or calculators. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided.

1. True or False.

   (a) _____ Every finite metric space embeds in $\mathbb{R}^n$ with the $\mathcal{L}^1$ metric, for some n.

   (b) _____ Every finite metric space embeds in $\mathbb{R}^n$ with the $\mathcal{L}^2$ metric, for some n.

   (c) _____ Every finite metric space embeds in $\mathbb{R}^n$ with the $\mathcal{L}^\infty$ metric, for some n.

2. What is the best known upper bound on the competitiveness for the 3-server problem in a general metric space? Who proved it? (Two names.)

3. What is the best known lower bound on the competitiveness for the 3-server problem in a general metric space? Who proved it? (Two names.)

4. What is the least competitiveness of any randomized trackless algorithm for the k-paging problem, for general k? What is that algorithm? (Just the name of the algorithm will do.)

5. The work function algorithm is known to be _____-competitive for any metrical task system with n states.

6. Give the definition of a metric space.

7. Give the definition of a metrical task system.

8. Give the definition of the competitiveness of a deterministic online algorithm for a metrical task system.

9. Give the definition of the competitiveness of a randomized online algorithm for a metrical task system against the oblivious adversary.

10. Our cow is looking for the hole in the fence, and adopts the following randomized strategy.

    - With probability $\frac{1}{2}$, she moves one meter to the left; otherwise, she moves one meter to the right.
    - If she has not found the hole, she returns to the previous step.

    Assume that the hole is just one meter to the left of the starting point. Therefore, she could (if lucky) get there in one step.

    (a) Prove that the the probability is 1 that the cow will eventually find the hole.

    (b) Prove that the expected waiting time for the cow to find the hole is infinite.

11. You may use randomization to pick either A, B, or C. Your opponent (who is oblivious) must pick either D, E, or F. The Cost Matrix below shows your cost for each of the nine possible situations. Find the randomized strategy which minimizes your expected cost.

	D	E	F
A	1	5	0
B	4	2	5
C	4	2	4

12. Let M be the metric space of four points, $\{x, y, z, w\}$, whose metric is given by Table 1. Consider the 2-server problem in M, where the initial configuration is $\{x, y\}$. For the request sequence $\varrho = zwy$, fill in the sequence of **offset** functions in Table 2.

	x	y	z	w
x	0	2	3	5
y	2	0	4	3
z	3	4	0	4
w	5	3	4	0

Table 1: The metric

	$\{x, y\}$	$\{x, z\}$	$\{x, w\}$	$\{y, z\}$	$\{y, w\}$	$\{z, w\}$
ω^0						
ω^1						
ω^2						
ω^3						

Table 2: The sequence of offset functions

13. Prove that *Move-to-Front* is 2-competitive for the list accessing problem.

14. Given a metric space M that has at least $k + 1$ points, prove that there is no online algorithm for the k-server problem in M whose competitiveness is less than k.

15. Consider the following metrical task system. There are two states, which we call A and B, and the cost of moving from one to the other is 2. There are two tasks, which we call α and β. The cost matrix is as given.

	A	B
α	0	1
β	2	0

    (a) Draw an appropriate red-black game graph.

    (b) Compute the deterministic competitiveness of this metrical task system. Indicate (upper bound) potentials and critical edges in the red-black game graph.