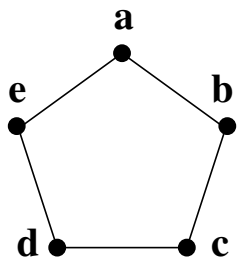
Homework CS 789 Spring 2011, March 31, 2011

Exercise 1 *Prove that every metric space of order n (that means, which has n points) embeds in* R^{n-1} *with the* $\mathcal{L}_\infty$ *metric.*

Exercise 2 *Prove that every metric space of four or fewer points embeds in the Manhattan plane.*

Exercise 3 *If G is a connected graph, the set of vertices of G is a metric space, using the shortest path length as the metric. The graph shown here is $K_{2,3}$. Any graph theorist would instantly tell you that $K_{2,3}$ can be embedded in the plane, as we show here, since those embeddings ignores distances.*



	a	b	c	d	e
a	0	1	2	2	1
b	1	0	1	2	2
c	2	1	0	1	2
d	2	2	1	0	1
e	1	2	2	1	0

Metric space C_5

However, if we use the shortest path metric, C_5 cannot be embedded in either the Euclidean or the Manhattan plane. Prove that C_5 can be embedded in R^5 with the $\mathcal{L}_1$ metric.