Homework CS 789 Spring 2011, April 5, 2011

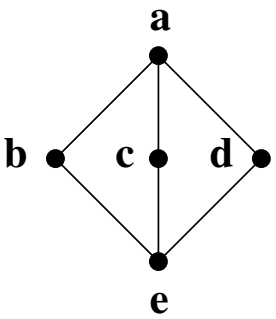
Exercise 1 *For any given n, let d be the $\mathcal{L}_\infty$ metric on R^n.*

Suppose that $x, y, z \in R^n$. Prove that there exists a point $w \in R^n$ such that the following equations all hold:

$$d(x,y) = d(x,w) + d(w,y) \quad (1)$$
$$d(x,z) = d(x,w) + d(w,z) \quad (2)$$
$$d(y,z) = d(y,w) + d(w,z) \quad (3)$$

Exercise 2

If G is a connected graph, the set of vertices of G is a metric space, using the shortest path length as the metric. The graph shown here is $K_{2,3}$. Any graph theorist would instantly tell you that $K_{2,3}$ can be embedded in the plane, as we show here, since that embeddings ignores distances.



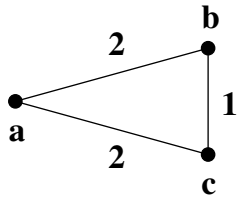
	a	b	c	d	e
a	0	1	1	1	2
b	1	0	2	2	1
c	1	2	0	2	1
d	1	2	2	0	1
e	2	1	1	1	0

Metric space $K_{2,3}$

However, if we use the shortest path metric, $K_{2,3}$ can be embedded in neither the Euclidean nor the Manhattan plane.

Prove that that $K_{2,3}$ cannot be embedded in R^n with the $\mathcal{L}_1$ metric for any n.

If G is a weighted graph with positive weights, then G is a metric space if we use the *minimum weight path* distance between vertices of G. The weighted graph below defines a three point metric space.



Exercise 3 *Find the randomized competitiveness of the 2-server problem on the three point metric space shown.*

Carsten Lund and Nick Reingold claimed (I believed at the time) that the randomized competitiveness for any three point metric space is less than $\frac{e}{e-1} \approx 1.582$. The last time I spoke with Reingold, he told me they never finished their study of three point metric spaces. Nevertheless, I believe the following:

1. The randomized competitiveness of the 2-server problem on any three point metric space is strictly less than $\frac{e}{e-1}$.
2. For any positive ϵ, there is a three point metric space whose randomized competitiveness for the 2-server problem is greater than $\frac{e}{e-1} - \epsilon$.