Homework CS 789 Spring 2011, April 12, 2011

The following diagram gives the red-black game equivalent to the Ice Cream problem.

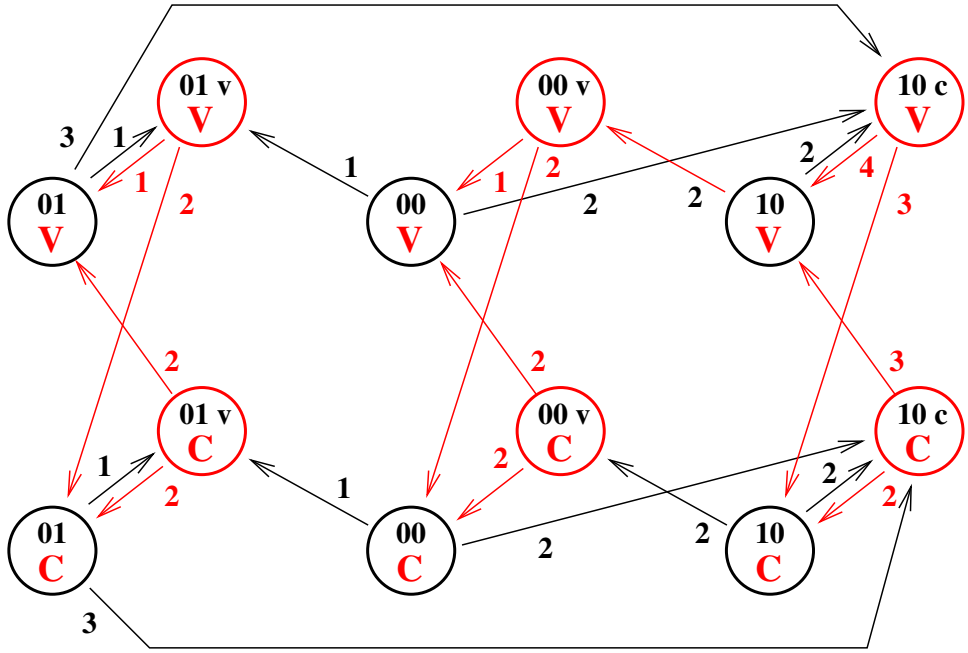


Figure 1: The Red-Black Game Equivalent to the Ice Cream Problem

Exercise 1 *Find the deterministic competitiveness of the Ice Cream Problem.*

Exercise 2 *In class, I told you that you can find the randomized competitiveness of the Ice Cream problem against the oblivious adversary by only considering three distributions for the algorithm:* V, $(1-p)V + pV$ *for some* $0 < p < 1$, *and* C.

Using those three distributions, find the competitiveness. You should pick p *so as to minimize the competitiveness.*

Hint: p is not equal to $\frac{1}{2}$, but it is a fairly simple fraction. Of course, C is also a fraction.

In class, I told you that you can find the randomized competitiveness of the Ice Cream problem by only considering three distributions for the algorithm, namely the pure states V and C, and one mixed state, $(1-p)V + pC$, for some $0 < p < 1$.

I was wrong!

Exercise 3 *Compute the randomized competitiveness of the Ice Cream problem against the oblivious adversary.*

The actual randomized competitiveness of the Ice Cream problem, against the oblivious adversary, is slightly less than what you computed using that incorrect assumption. In fact, you need three mixed states, namely: $(1-p)V+pC$, $(1-q)V+qC$, and $(1-r)V+rC$, where $0 < p < q < r < 1$.

You will also need the pure state V, because that is the initial distribution.

Initially, the algorithm is in the pure state, V, and will stay there as long as all requests are for vanilla. But once chocolate has been requested, even just once, the algorithm will never again be in any pure state.

Hint: When you draw the diagram, you will find five cycles. Using those five cycles, you can obtain four linear equations in four unknowns, namely C, p, q, and r.

Of course, these four numbers are all rational.

In class, I told you that if the offset function is a cone, namely is supported by a single configuration X, then the distribution of the optimal randomized algorithm is X, *i.e.,*it is at X with probability 1. Let us call that the "definitely at the cone point" property. (We will have to think of some shorter name!)

This "definitely at the cone point" property is true for lots of metrical task systems, including paging and the server problem, but is false for the ice cream problem.

Exercise 4 *What property of a metrical task system ensures that the "definitely at the cone point" property holds?*

There could be more than one correct answer to this exercise.