The Trackless Server Problem

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1 Trackless Work Functions

We define *trackless work functions* for the k-server problem as follows.

2 The Trackless Work Function Algorithm

We define the *trackless work function algorithm* for the k-server problem as follows. After receiving a request, the algorithm TWFA moves one server to serve the request, in such a way as to minimum the sum of its cost and the value of the trackless work function on its new configuration. If there are multiple minimal choices, TWFA makes one of those choices arbitrarily. (In our analysis, we shall assume that the adversary makes the choice.)

3 Results

Define $\mathcal{M}_d$ to be the class of all metric spaces whose distances are integers which do not exceed d. Thus, for example, $\mathcal{M}_1$ is the class of all uniform spaces.

Using a computer program, we have verified the following:

1. The competitiveness of TWFA for the 2-server problem in $\mathcal{M}_d$ is 3, for $3 \leq d \leq 6$.
2. The trackless competitiveness of the 2-server problem in $\mathcal{M}_d$ is C_d, where C_d is given below for all $d \leq 5$:

$$\begin{aligned} C_1 &= 2 \\ C_2 &= 2 \\ C_3 &= \frac{11}{5} = 2.2 \\ C_4 &= \frac{55}{24} \approx 2.29167 \\ C_5 &= \frac{7}{3} \approx 2.33333 \end{aligned}$$

4 Lower Bound for TWFA

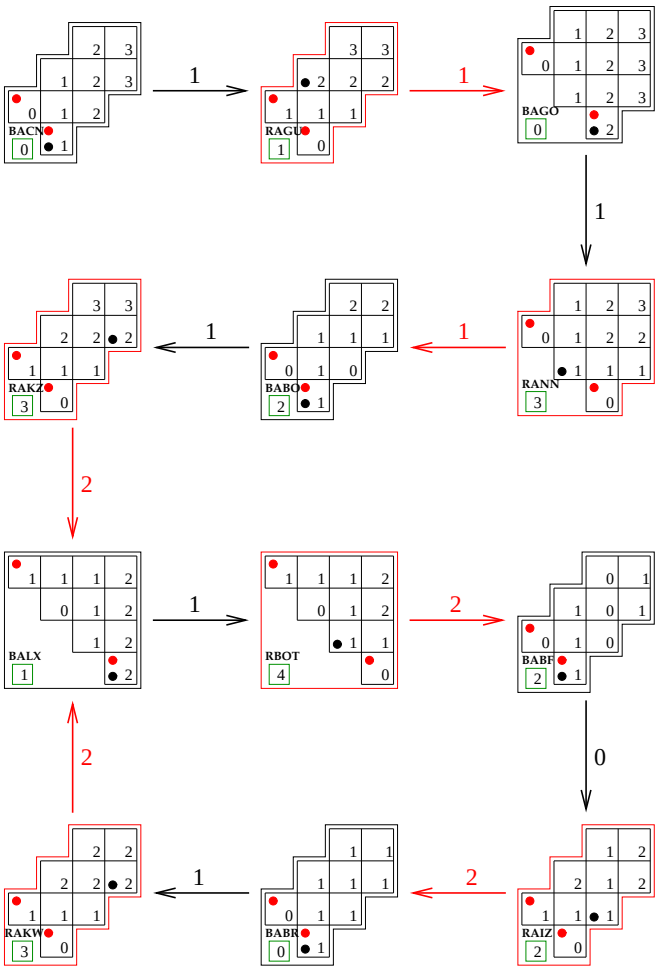


Figure 1: Lower Bound Proof for TWFA

The lower bound is proved as follows. Let $\mathcal{G}$ be the bipartite graph consisting of all configurations which are reachable, if TWFA is used.

In Figure 1, we show a portion of $\mathcal{G}$. algorithm costs are shown in red, while adversary costs are shown in black.

Let α be any cycle in $\mathcal{G}$. Let $cost_{\text{OPT}}(\alpha)$ be the sum of the amortized optimal costs of α, and let $cost_{TWFA}$ be the sum of the algorithm costs of α. If TWFA is C-competitive, then $cost_{TWFA}(\alpha) \leq C \cdot cost_{\text{OPT}}(\alpha)$.

In Figure 1, the cycle consisting of six configurations in the center of the figure has optimal cost 2 and algorithm cost 6. This proves a lower bound of 3 for the competitiveness of TWFA on $\mathcal{M}_3$. If we insist on a proof that begins with a matching configuration, consider the path α from BACN to BALX then around the cycle n times. Then $cost_{TWFA}(\alpha) = 6n + 4$, while $cost_{\text{OPT}}(\alpha) = 2n + 3$. As n increases, the limiting competitive ratio is 3.

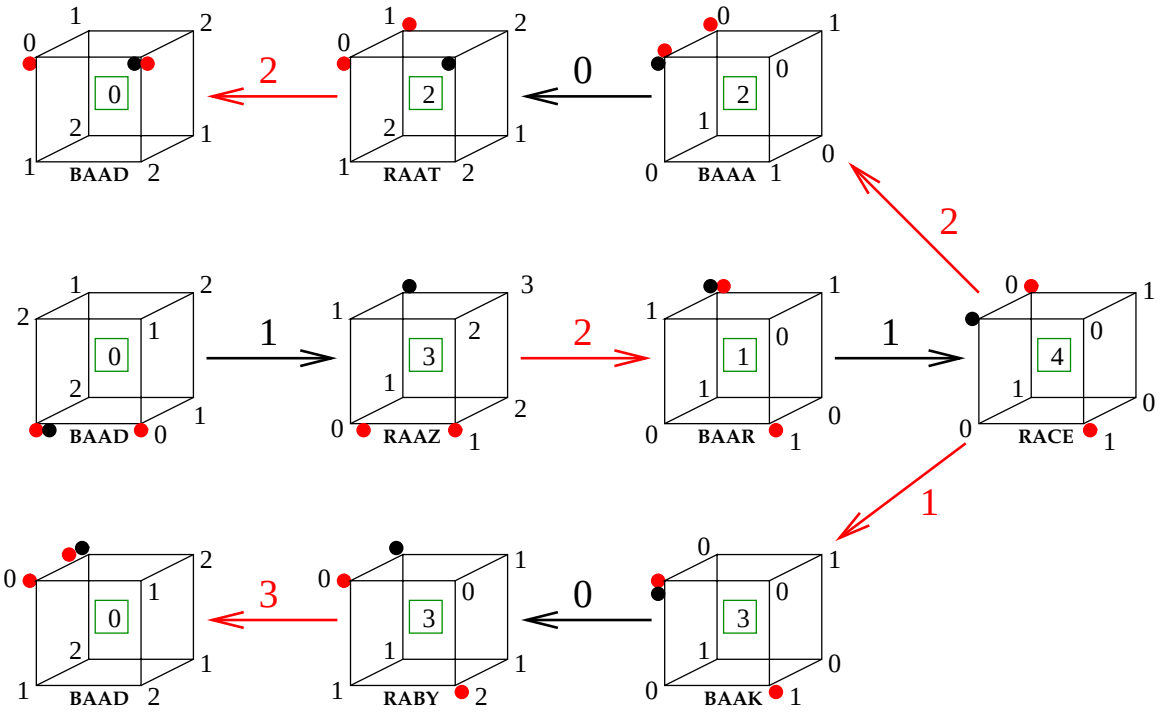


Figure 2: Lower Bound Proof for TWFA

The lower bound of 3 is also proved if the metric space is a 3-cube. There are two overlapping cycles, each consisting of three requests, shown in Figure 2. The configurations shown are reachable from any start configuration, in a manner similar to that shown above.

5 Upper Bound for TWFA

Computer calculations have established that TWFA is 3-competitive for a generic metric space where all distances are integers at most 6. At the present, we have no proof.

6 Lower Bound for the Trackless 2-Server Problem

Computer calclations have established that no trackless online algorithm for the 2-server problem can have competitiveness less that $\frac{7}{3}$. This lower bound was found by restricting the problem to the class $\mathcal{M}_5$. The certificate for this lower bound contains a graph with approximately two hundred thousand nodes.

Interestingly, we obtain the same lower bound if we restrict to the class $\mathcal{M}_{5,10}$ consisting of all metric spaces with integer distances which are at most 5, with the restriction that the perimeter of any triangle is at most 10. At the present time, our certificate contains a graph of approximately one thousand nodes, which is still too many for a hand proof. We hope to reduce the size of the certificate. We do not conjecture that the true competitiveness of the problem is $\frac{7}{3}$.